

Aggressive Boards and CEO Turnover*

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Abstract

This study investigates a communication game between a CEO and a board of directors where the CEO's career concerns can potentially impede value-increasing informative communication. By adopting a policy of aggressive boards (excessive replacement), shareholders can facilitate communication between the CEO and the board. The results are in contrast to the multitude of models which generally find that management-friendly boards improve communication, and help to explain empirical results concerning CEO turnover. The results also provide the following novel predictions concerning variation in CEO turnover: (i) there is greater CEO turnover in firms or industries where CEO performance is relatively more difficult to assess; (ii) the board is more aggressive in their replacement of the CEO in industries or firms where the board's advisory role is more salient; and (iii) there is comparatively less CEO turnover in firms or industries where the variance of CEO talent is high.

Keywords: Corporate governance, advising, CEO replacement, communication, CEO turnover, board independence.

JEL classification: C72, D82, D83, G34, M41, M51.

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1 Introduction

A natural tension arises between the CEO of a firm and its board of directors, insofar as the board must sometimes take disciplinary measures on the top executives while simultaneously helping through guidance. One of the board's primary responsibilities is to decide whether to replace or retain the CEO (Lorsch and MacIver (1989), Laux (2014)). The board also serves to provide the CEO with guidance and advice concerning the firm's direction, thus benefiting the CEO and shareholders. As Mace et al. (1971) notes, "directors serve as a source of advice and counsel, serve as some sort of discipline, and act in crisis situations" (p. 178). Survey evidence also documents that board members overwhelmingly believe that they help shape the firm's strategic direction (Demb and Neubauer (1992)). However, the CEO and top executives control the non-public information that the board receives.¹ Consequently, the CEO wishes, and is often able, to conceal negative information from the board. The allure to manipulate information places the CEO in an unfortunate predicament: she stands to benefit from the board's guidance and expertise, but in doing so she must communicate potentially unfavorable information about the firm's current operations, which consequently lowers the board's assessment of her ability. Indeed, it has been a significant concern among U.S. public firms that CEOs often fail to effectively communicate with boards by concealing negative information from board members, as exemplified by the infamous cases of Enron and Worldcom.²

To further illustrate the dilemma an incumbent manager faces, consider a CEO who observes preliminary information regarding a project that she has been tackling (such as the development of a new product). The preliminary information is negative and hence the CEO is confronted with a problem regarding the best path forward for the project. The CEO can honestly reveal the problem to the board, and in turn she receives the board's expert advice concerning the most viable solution. This allows the CEO to take the best action going forward. However, if the CEO communicates honestly, the board infers that she is of a low ability, considering that her project was not successful, and this may affect the board's decision to replace the CEO. Alternatively, the CEO can overstate the performance

¹This has also been noted by Song and Thakor (2006) and Adams and Ferreira (2007), and has been referenced in the news: "[directors] depend largely on the chief executive and the company's management for information" (*The Economist*, March 31, 2001). Moreover, as Jensen (1993) notes: "The CEO most always determines the agenda and the information given to the board" (p. 864).

²Other examples include the CEO of Kmart misleading the board of directors regarding supplier payments in 2001; see "Former CEO misled board, Kmart says," *Bloomberg News*, February 25, 2003. Moreover, the CEO of Braidy Industries, an aluminum mill company, reportedly overstated the company's financial prospects; see "Former Aluminum Mill CEO Misled Investors, Board," *Associated Press*, April 27, 2020. Larcker and Tayan (2016) discuss cases of CEOs lying to board members about personal information. For empirical evidence on CEO turnover and accounting misreporting, see Hennes et al. (2007).

of her project thus far (such as conveying a milder problem), but then the solution offered by the board will not be helpful for her. Conversely, the manager would have no such inhibitions in truthfully communicating good news, and receiving advice on the best action to take following a more successful project (e.g., increasing investment). Overall, the board's guidance is effective as long as the CEO honestly communicates with the board regarding the current status of the firm, however the CEO's reputational concerns may compel her to conceal or misrepresent negative information.

In this paper, we investigate this interdependency between the advisory and disciplinary roles of the board in a communication game with a CEO. The CEO aims to increase the value of the firm, while also receiving personal benefits from staying in power. The CEO observes private information θ regarding her ability or productivity at the firm and then sends the board a report $\hat{\theta}$ concerning her private information. This report captures, for example, the status of projects the CEO has been undertaking during her tenure. After observing the report $\hat{\theta}$ from the CEO, the board provides advice that can be value-increasing for the firm. In particular, the board's advice is only helpful insofar as the CEO was honest with the board (i.e., $\hat{\theta} = \theta$). However, when unfit for the firm, the CEO is tempted to distort the report $\hat{\theta}$ upwards in an attempt to preserve her position. The board then observes the firm's output y (i.e., a performance measure such as earnings) and decides whether to replace or retain the CEO. Prior to the beginning of the game described above, shareholders, who aim to maximize the firm value, determine the optimal board policy on the replacement of the CEO. Shareholders can set a friendly board, for example, by making CEO removal difficult or through appointing lenient directors. Conversely, shareholders can design a strict or aggressive board by allowing the board to swiftly replace the CEO. By examining the interplay between advising and replacement, we determine the shareholders' optimal board policy.

As the main result of this paper, we find that shareholders often prefer the board to be aggressive because *an aggressive board enhances truthful communication*. This result is in contrast to the multitude of models which find a benefit to management-friendly boards, such as Almazan and Suarez (2003), Adams and Ferreira (2007), Harris and Raviv (2008), Laux (2008), Casamatta and Guembel (2010), Inderst and Mueller (2010), Dow (2013), and Chakraborty and Yilmaz (2017).³ As we explain below, our results differ from these studies, which separately consider advising or replacement, by highlighting how the board's replacement decision interacts with its advisory role.

The primary economic force that drives aggressive boards to be optimal is because strict

³Hermalin and Weisbach (1998) and Warther (1998) find passive boards as the equilibrium outcome arising from CEO influence over the board.

replacement practices induce greater truthful communication by disciplining CEOs of low ability. As described earlier, the board decides whether to retain the CEO after observing her report $\hat{\theta}$ and output y . A weak CEO has an incentive to inflate her ability in the report $\hat{\theta}$, since a truthful, pessimistic report convinces the board of the CEO's incompetence and leads to her certain replacement. Thus, the low-ability CEO, who receives personal benefits from remaining in power, often exaggerates her productivity in order to have a chance at retention, reducing the effectiveness of the board's advice and the value of the firm. This misreporting incentive can, however, be weakened under an aggressive board and its very strict retention policy. An aggressive board is inclined to replace the CEO and thus sets a high standard for retention on the outcome y , even if the report $\hat{\theta}$ is optimistic enough. With the likelihood of retention reduced, the low-type CEO finds misreporting less attractive and is more inclined to communicate truthfully with the board. This *disciplining effect* is the primary economic force for shareholders to increase the aggressiveness of the board.

However, this more rigorous screening and consequent improvement in truthful communication comes at the cost of distorted retention decisions. Since some weak CEOs inflate their reports and pretend to be high productivity types, a promising report $\hat{\theta}$ does not necessarily imply that the CEO is satisfactorily productive. As a result, the board relies on the secondary piece of information, output y , in evaluating the ability of the CEO and determining the retention decision. An aggressive board is biased towards excessively replacing the CEO and thus sets a suboptimally strict standard for output y in the retention decision. This suboptimality in the retention decision is referred to as the *distortion effect* of an aggressive board. The optimal board is determined by balancing the abovementioned two conflicting forces. Our main results characterize the conditions under which the disciplining effect dominates the distortion effect.

The optimality of aggressive boards emerges from the following interesting, paradoxical effect of board aggressiveness and its interaction with CEO replacement. In the presence of a replacement decision, the CEO and shareholders are naturally misaligned in their preference for retention—the CEO would like to retain power at the expense of firm value while shareholders and the board wish to remove poor CEOs. Aggressiveness *amplifies* this misalignment by encouraging the board to remove the CEO more frequently. This intensified misalignment in the board's preferences, paradoxically, helps management *coordinate* with the board, essentially depriving the CEO of the chance at survival. Consequently, an aggressive board is able to collapse all of the CEO's expected payoff on the benefit she receives from informative communication with the board, rather than on her continuation within the firm. In other words, misalignment in preferences, widened by aggressiveness, helps to align incentives—both the CEO and the board seek to maximally benefit from effective

communication.

In contrast, strategic communication models without CEO replacement often find that misalignment in preferences *negatively* impacts truthful communication (e.g., Chakraborty and Yilmaz (2017)). In these models, greater misalignment does not shift the manager's objective towards maximizing firm value because of the lack of the CEO replacement decision, which converts board aggressiveness into CEO disciplining. In models without CEO retention, a management-unfriendly board can be *more* punitive to truthful CEOs than to miscommunicating ones. Indeed, in the model of Adams and Ferreira (2007), the board intervenes more often following truthful communication, and as the board becomes less management-friendly, CEOs are discouraged to truthfully communicate with the board.⁴ Conversely, it is the key distinction under a replacement decision in our setting that intensified management-unfriendliness, relative to neutrality, penalizes *only* miscommunicating CEOs and *does not* entail an additional negative effect on CEOs who truthfully communicate with the board.⁵ More specifically, we find that an aggressive board adopts an equally strict replacement policy as a neutral board for truthful but unfit CEOs, whereas the policy towards misreporting CEOs is harsher under an aggressive board than a neutral one. In this sense, an aggressive board is more punitive to misreporting CEOs than a neutral board, while not any more punitive towards truthful low-type CEOs. Hence, the interplay between the disciplinary (i.e., replacement) and advisory roles of the board critically contribute to our novel finding that aggressive boards can be optimal for communication.⁶ We discuss this further in Section 6.1.

We note that it is intuitive that friendly boards could also facilitate truthful communication in our model as well: when the board is very friendly, the CEO is under no threat of eviction and therefore has little incentive to distort her communication to the board. However, the improvement in communication comes at the cost of severe distortion in the retention decision. While a friendly board can elicit truthful communication from a low ability CEO through an overly lenient retention policy, shareholders and the firm will then be stuck with this low type manager in the future, thus harming *future* firm value. An aggressive board is more efficient in this regard: although an aggressive board often dismisses

⁴See footnote 11 for details. See also Section B.3 of Adams and Ferreira (2007). While the replacement decision works as one of the primary economic forces that drive our results, the binary nature of replacement contributes to tractability and permits a clean characterization of the solution (discussed further in Internet Appendix D).

⁵To be more precise, this effect applies only to low-type CEOs. In equilibrium, high-type CEOs do not misreport their types in our setting.

⁶We note that another distinguishing feature of our model is the presence of an additional signal regarding the CEO's private information after communication with the board. This additional information allows the board's strategies to be represented as cutoff strategies, which is a novel addition to the literature on board-CEO communication.

talented managers contrarily to the benefit of shareholders, the board removes low types more often than high types, improving CEO quality.

Two additional notable features emerge in this analysis. First, as mentioned above, the board employs a two-prong replacement strategy, first in the communication and then in the observed output. Even if the manager's initial report $\hat{\theta}$ is high enough, the board removes the CEO when observed output y is below the standard. This replacement policy is consistent with the findings of Cornelli et al. (2013), who show that soft, non-verifiable information regarding the CEO's ability (i.e., report $\hat{\theta}$) is a salient factor in the board's CEO replacement decision. Likewise, the secondary criterion based on the output performance y is consistent with a number of empirical studies which find that CEOs are more likely to be removed following poor performance.⁷

As the second feature, the manager's optimal reporting strategy is *non-monotone* in her ability. As discussed above, weak CEOs often inflate their reports to conceal information that would otherwise result in their dismissal. Consequently, misreporting CEOs sometimes inflate their reports even higher than types that are above the standard (as shown in Figure 5 later). Relatedly, we also find that truthful communication is non-monotone in type: CEOs of very high or very low ability report truthfully, while intermediate types inflate their reports.

In addition to the main result above, our model provides novel insights and predictions concerning variation in the frequency or likelihood of CEO turnover. One new prediction which emerges is that there should be greater CEO turnover in industries or firms where CEO performance is more difficult to assess. This novel implication helps to explain the findings of Jenter and Kanaan (2015), who find that CEO turnover is higher during periods of heightened market uncertainty.⁸ Moreover, the results have implications concerning CEO turnover in firms or industries that rely on or can benefit more from board advising. For example, firms with multiple segments, dispersed operations, complex financial structures, or firms that frequently face major corporate decisions can benefit more from board expertise (Boone et al. (2007), Linck et al. (2008)). The results predict that CEO turnover is higher in such firms or industries where the board's advisory role is more salient. Additionally, with regard to the dispersion of talent in the CEO labor market distribution, we find that the board is more aggressive in their replacement of the CEO when the variance of CEO talent

⁷For the relation between CEO turnover and firm performance, see Warner et al. (1988), Kaplan (1994), Denis and Denis (1995), Huson et al. (2001), Huson et al. (2004), and Faleye et al. (2011), among others.

⁸In addition, Lel et al. (2014) and Gao et al. (2017) find that private firms, which typically have closer monitoring of executives by investors, replace their CEOs less often than public firms. Cornelli and Karakaş (2015) similarly find that CEO turnover is lower in firms undergoing leveraged buyouts, where private equity firms are more involved in the firm's operations, allowing them to more easily assess the CEO's ability from observed outcomes.

is low.⁹ All of these predictions concern variation in CEO turnover, which has not been fully explored in the literature. Our analysis may therefore help to provide guidance for further empirical investigation. These predictions, as well as others, are more thoroughly discussed in Section 5.

We note that the model incorporates a signaling structure whereby the CEO, as the sender, submits a message regarding her ability to the board, as the receiver. We therefore need to apply appropriate equilibrium selection. Notably, we obtain a unique equilibrium after the selection even though this setting often violates standard monotonicity assumptions. In particular, since truthful communication has a special meaning in our model, interesting but ill-behaved *productivity reversal* often occurs in the following sense: a high type with a misreport may become less productive than a low type with a truthful report.¹⁰ Due to the absence of natural monotonicity assumptions, we unavoidably need to handle pathological, non-monotonic actions from the receiver (i.e., the board)—an action of the receiver is a function of the output in our setting. As a methodological contribution, we succeed in constructing a method that allows us to evaluate arbitrary functions as if they are regular and well-behaved (see Lemma 12 in Internet Appendix E). The methods developed here may be more broadly applicable in solving signaling or information-transmission models which similarly involve non-monotonic strategies.

1.1 Related literature

The extant theoretical literature investigating equilibrium models of board independence generally emphasize the efficacy of passive boards for the communication between the board and CEO. We first discuss the model in relation to the literature that examines the advising capacity of boards. Adams and Ferreira (2007) consider a model where a privately informed CEO can receive value-increasing advice from the board after truthfully revealing her private

⁹Empirical studies have also documented that poor industry performance increases the likelihood of forced CEO turnover (e.g., Kaplan and Minton (2012), Jenter and Kanaan (2015)). In the context of the model, poor industry performance may be interpreted as a high density of poorly performing managers. In this case, a high concentration of weak managers leads to aggressive boards, and thus increased CEO turnover in equilibrium.

¹⁰The problem of productivity reversal always occurs with continuously many types and sometimes even with two types (before we uniquely identify an equilibrium where irregularities are already eliminated). With the productivity reversal, higher outputs do not necessarily increase the likelihood of higher-ability types. Consequently, it becomes intractable to characterize the optimal retention policy for the board. (As we will see later, optimal retention policies are usually a cutoff rule.) See the proofs of Theorem 1 and Lemma 10 for more details. Besides the productivity reversal, it is another anomaly that our model does not satisfy the single-crossing condition between type θ and report $\hat{\theta}$ because, once again, of the special importance of the truthful report.

information.¹¹ Harris and Raviv (2005, 2008, 2010) study models of strategic communication between two privately informed parties (e.g., the manager and board; Harris and Raviv (2008)) in which one of the two parties has decision-making authority. Relatedly, Baldenius et al. (2014) and Chakraborty and Yilmaz (2017) consider information transmission between the board and the CEO.¹² In contrast to these papers, our model allows the board to replace the CEO so that weak prospects from the CEO can result in the termination of her tenure.¹³ Moreover, in the present setting, the board can observe an additional signal concerning the CEO's type between the board's advisement and replacement decision, which improves the effectiveness in eliminating unfit CEOs.

Several papers considering CEO replacement have found benefits of management-friendly boards. Almazan and Suarez (2003) investigate optimal board design where the CEO has veto authority; however, they assume that there is no asymmetric information between the CEO and the board, whereas this is a central feature of the present model. Laux (2008) and Inderst and Mueller (2010) consider CEO replacement under private information. Although communication is allowed in these settings, our main focus is on the advising nature of com-

¹¹Adams and Ferreira (2007) allow the CEO to withhold her private information, as an alternative to fully revealing it. In their setup, a truthful CEO suffers more than one who withholds information from the board, because truthful revelation involves a hold-up problem and allows the board to more easily intervene in the firm's operations. This tendency for intervention worsens as the board becomes more independent. Notably, Adams and Ferreira (2007) do not incorporate CEO ability into their model, and thus the CEO's private information has no correlation with her ability. In contrast, in our setting, the manager's type reflects her ability. Consequently, in the case of a below-average CEO, she is treated more poorly (i.e., replaced) with truthful revelation than with misreporting. This feature itself is similar to Adams and Ferreira (2007), but this gap in the board's disciplining narrows as the board becomes more aggressive, unlike Adams and Ferreira (2007), by actively replacing CEOs with inflated messages (Section 3). This disparate effect of board aggressiveness (or independence) is the main distinction that ultimately leads to the optimality of aggressive boards in our model. We illustrate this contrast further in Section 6.1.

¹²Baldenius et al. (2014) study the trade-off between the monitoring and advisory roles of the board in a setting where shareholders determine the weights between these two activities. The authors find that shareholders prefer centralized power by the board when the CEO is known to have a large bias, and prefer to delegate the decision-making authority to the CEO when the CEO's bias is small. Chakraborty and Yilmaz (2017) consider a communication game where shareholders allow the board to be biased so as to be aligned with the CEO's preference. The authors calculate the optimal alignment (friendliness) of the board in a setting where aligned preferences improve the communication between the CEO and the board. These studies primarily focus on the strategic communication between the CEO and the board, while abstracting away the board's disciplinary capacity to eliminate unfit CEOs. In this paper, we analyze how the board's disciplinary capacity, together with its advisory capacity, affects the communication between the CEO and the board. (See Section 6.1.) Also, while their models allow the decision of the board to be delegated to the CEO, in this paper we naturally prohibit the delegation of the retention decision of the CEO to the CEO herself.

¹³Relatedly, Song and Thakor (2006) examine a model of strategic communication where both the CEO and the board care about their perceived talent. Also, a recent paper by Gregor and Michaeli (2020) considers board-CEO communication in a Bayesian persuasion setting and investigates properties of optimal misalignment of the board. Our primary objective is to examine the interdependency between advising and replacement, whereas replacement and turnover are not part of these models.

munication and its relation with replacement, whereas advising is absent in Laux (2008) and Inderst and Mueller (2010). Crémer (1995) considers a principal-agent model with replacement and finds that the principal may prefer to weaken a monitoring technology. In Crémer (1995), the better monitoring technology can uncover the agent’s private information and gives her a better chance of survival, which undermines the incentive provided in the labor contract. In contrast, in our model, shareholders prefer inefficient replacement in order to uncover the CEO’s private information and foster value-increasing communication. Our setting is also related to governance models which incorporate learning of the CEO’s ability (e.g., Dominguez-Martinez et al. (2008), Hermalin and Weisbach (2012)). We complement this literature by investigating the interdependency between CEO replacement and the advisory role of boards.¹⁴

There is also a literature that studies the cost of lying in strategic communication, which, in the context of the present paper, appears as decreased productivity following miscommunication. Kartik et al. (2007) and Kartik (2009) extend the Crawford and Sobel (1982) cheap talk framework by introducing a differentiable lying cost. Although the present setting can be seen as a model of misreporting costs, the receiver (i.e., the board) in our model observes an additional signal after observing the sender’s message but before the receiver’s retention decision.¹⁵ In addition, while Kartik et al. (2007) and Kartik (2009) explore the properties of strategic communication itself, our primary focus is on how the board and shareholders handle the miscommunication problem of the CEO when misreporting is costly for all parties.

The paper is structured as follows. The next section introduces the model and Section 3 presents the equilibrium of the baseline setting. Section 4 solves the model under continuously many types, while Section 5 provides empirical predictions. Section 6 discusses possibilities of alternative formulations. The final section concludes. All proofs are relegated to Appendix B unless otherwise specified.

¹⁴Laux (2017) considers a replacement model where the principal can provide costly support for the agent to increase the chance of a positive outcome, which relates to our notion of value-increasing advice in the present setting. Laux (2017), however, does not have an additional variable that signifies friendliness, unlike the present study. In addition, among other differences, the CEO in our setting has private information over her type and we consider the effect of replacement on communication.

¹⁵Also, the misreporting cost in the current paper is discontinuous and does not fit the framework of Kartik et al. (2007) and Kartik (2009). In this paper, the misreporting cost $C(\hat{\theta}, \theta) = \mathbf{1}_{\{\hat{\theta} = \theta\}}$ is discontinuous in θ , while both Kartik et al. (2007) and Kartik (2009) require $C(\hat{\theta}, \theta)$ to be differentiable in θ .

2 The model

We study a two-period model where the board of a firm decides whether to continue employing the current CEO after observing information concerning her ability in the first period. The board hires a CEO at the beginning of each period if the position is vacant. The CEO in period t learns her productivity $\theta^t \in \Theta$ in this firm (or her fitness to this firm). The type is perfectly persistent so that the period-2 type θ^2 is identical to the period-1 type θ^1 if the board decides to retain the manager at the end of period 1. Otherwise, θ^2 is drawn independently of θ^1 . In Section 3, we study a two-type model with $\Theta = \{\theta_H, \theta_L\}$ where $\theta^t = \theta_H$ occurs with probability $\pi \in (0, 1)$. We also study continuous types in Section 4 (and Internet Appendix C), where the type space is $\Theta = (\underline{\theta}, \bar{\theta})$ with $-\infty \leq \underline{\theta} < \bar{\theta} \leq +\infty$. The distribution of the type θ^t has a probability density function $g(\theta)$, which is positive and continuous on the support $(\underline{\theta}, \bar{\theta})$. The expected value $\mathbb{E}[\theta]$ is finite.

After learning her type θ^t , the manager then sends a report $\hat{\theta}^t \in \Theta$ to the board. We assume that the CEO simply reports her type to the board; however, the results would be qualitatively impervious to assuming that the CEO rather reports a performance signal about a current project that is correlated with her type. We seek to model the essence that the CEO reports a specific concern or problem to the board, which conveys implications about her type; the simplest modeling assumption that continues to preserve the economic insights is where the CEO directly reports her type to the board.

The board then advises the manager based on the report $\hat{\theta}^t$. The board is composed of individuals who are adept at solving particular problems, and can offer valuable guidance to the manager. Once the board receives the CEO's report, $\hat{\theta}^t$, it invests time to determine the appropriate course of action, or the state $\omega^t(\hat{\theta}^t)$, and transmits this to the CEO. The state $\omega^t(\hat{\theta}^t)$ can be thought of as the ideal solution or direction for the firm given that the CEO's report $\hat{\theta}^t$ correctly reflects her and her firm's status. Following Adams and Ferreira (2007), we assume that the board's advice is effective only when the CEO truthfully reports her type (i.e., $\hat{\theta}^t = \theta^t$). Otherwise, the board learns an irrelevant state $\omega^t(\hat{\theta}^t)$ that is statistically independent of the relevant state $\omega^t(\theta^t)$. The state $\omega^t(\theta^t)$ is uniformly distributed between 0 and 1.¹⁶ The board's advisory behavior is always truthful and non-strategic.¹⁷ Misreporting

¹⁶The results hold for any non-degenerate continuous distribution of $\omega^t(\theta^t)$.

¹⁷This assumption is purely for simplicity and does not substantively affect the results. Alternatively, we can potentially allow the board to intentionally give erroneous advice. In this setting, the board has no strict incentive to give inaccurate advice to the CEO and thus truthful advice always survives as one of the optimal choices. However, the board may become indifferent between truthful advice and uninformative advice when the board assigns probability one, for example, on the event that the high-type report always comes from a low-type CEO (i.e., there can exist an equilibrium where both CEOs report $\hat{\theta} = \theta_L$, and therefore the message $\hat{\theta} = \theta_H$ is not observed on the equilibrium path). Such pathological cases can be ruled out with equilibrium selection criteria. To avoid additional complexity and to focus the analysis on the main

thus does not lead to helpful guidance as the solution or direction offered by the board is then not relevant for the current state of affairs. This assumption regarding the reporting stage mirrors that of Adams and Ferreira (2007); however, the key distinction is that we assume that the CEO's private information also conveys the CEO's type.

Given the board's advice, the manager chooses an action $a^t \in [0, 1]$ that affects cash flow y^t . After misreporting, the CEO is unable to set $a^t = \omega^t(\theta^t)$, and as a result, we assume that this miscommunication impacts firm value in the form of a loss d in cash flows.¹⁸ In contrast, after truthful communication, the CEO is always able to match her action a^t with the relevant state $\omega^t(\theta)$. We can therefore focus on the scenario where the CEO loses d from the cash flow in period t only after a misreport $\hat{\theta}^t \neq \theta^t$ in the same period. Hence, period t cash flow is given by:

$$y^t = \theta^t - d \cdot \mathbf{1}_{\{a^t \neq \omega^t(\theta^t)\}} + \varepsilon^t.$$

The cash flows can be thought of as the outcome of an action a^t , plus the effect of the CEO's ability on firm value, which in this case is the CEO's ability, plus zero-mean noise ε . We assume that the action a^t is not publicly observed; the results would not be qualitatively affected if a^t was observable. We capture the benefits of truthful communication through this parsimonious reduced-form representation so that we may focus the analysis on the manager's reporting strategy, the board's replacement strategy, and the shareholders' board policy.

The noise ε has mean 0 and follows a distribution $F(\varepsilon)$ with density $f(\varepsilon)$. The density is symmetric (i.e., $f(-\varepsilon) = f(\varepsilon)$), positive, and continuous on the real line. The density also satisfies the following version of the *monotone likelihood ratio property*: whenever $\theta > \theta'$,

$$L(y|\theta, \theta') = \frac{f(y - \theta)}{f(y - \theta')}$$

is continuous and increasing in y , ranging from 0 (at $y = -\infty$) to ∞ (at $y = \infty$). For example, normal distributions with mean 0 satisfy this requirement.

At the end of the first period, the board strategically decides whether to retain or remove the current manager after observing the CEO's message $\hat{\theta}^t$ and output y^t . Let x represent the retention decision in the first period: $x = 0$ indicates that the board has decided to replace the CEO, while $x = 1$ corresponds to retention. The board incurs the payoff of c if the CEO is replaced, in addition to the output y^t . That is, the payoff to the board in period

interactions of the model, we assume that the board's advice is truthful.

¹⁸Alternatively, an equivalent assumption is that cash flows are improved through truthful communication.

1 is $y^1 + (1 - x)c$, and the board's total payoff is

$$U_b = [y^1 + (1 - x)c] + y^2.$$

The parameter c is the cost, or subsidy, that the board receives upon replacing the CEO. To answer our central question, we allow the shareholders of the firm to choose the parameter c before the beginning of the first period to see which level of c endogenously emerges. By controlling c , the shareholders incentivize the board to be more aggressive or passive in their removal of the CEO. Shareholders simply seek to maximize the sum of the firm's return in two periods:

$$U_s = y^1 + y^2.$$

Removal can be costly for the board, in which case $c < 0$. A negative c can be thought of as the degree to which the CEO is entrenched in the firm, and so board members must spend more time and energy orchestrating her removal, such as arranging a lengthy takeover bid primarily for the purpose of removing the CEO. Conversely, shareholders can set $c > 0$. This corresponds to the notion that shareholders can approve board members who are predisposed to removing the CEO, or can hold board members more accountable for weak performance in the absence of disciplinary action, thus inducing excessive turnover (Taylor (2010)).¹⁹ This concept of a “negative entrenchment” level (positive c in this case) has also been empirically documented (see Taylor (2010)).²⁰ CEO removal can also be personally beneficial for board members; this captures the notion that board members have the opportunity to serve as CEO in the event of removal, and hence prefer removing the CEO more often (as empirically documented in Mobbs (2013)). We define $c < 0$ to correspond to the notion of board *friendliness*, or passivity. Similarly, when $c > 0$, we refer to this as board *aggressiveness*, or excessive replacement. (See Section 5 for additional interpretations of c .)

The period-1 manager's objective is to maximize the expected value of

$$[\chi + y^1] + x \cdot [\chi + y^2]$$

where $\chi > 0$ is a rent from staying in position as CEO for each period. This rent can signify,

¹⁹Taylor (2010) discusses that board members may prefer to remove the CEO excessively in order to protect their reputation or position on the board. His empirical results suggest that, in the context of this paper, $c > 0$ for large firms with stronger governance. The model here thus develops theoretical underpinnings to help explain this empirical finding.

²⁰Taylor (2010) finds that “the degree of entrenchment is significantly lower in recent years and is slightly negative in larger firms” (p. 2053).

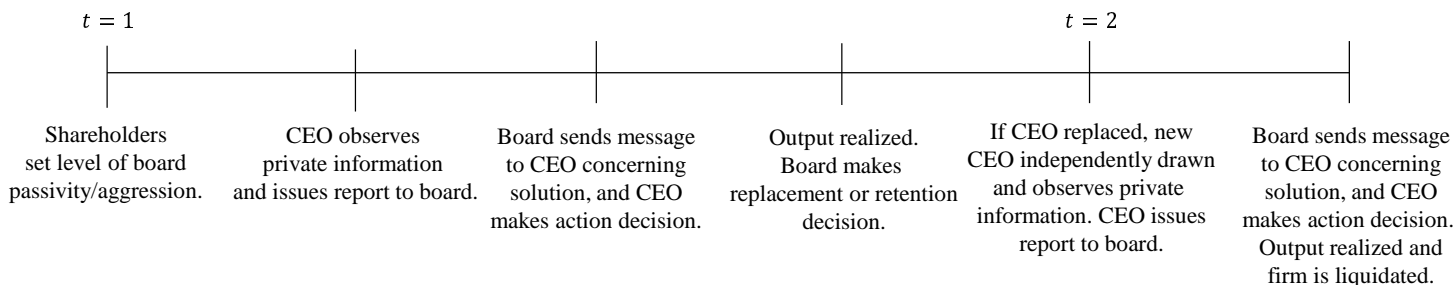


Figure 1: Timeline

for example, private benefits of control that the agent receives from being employed as CEO (Dewatripont and Tirole (1994), Dyck and Zingales (2004), Adams and Ferreira (2007)).²¹ The output y^t appears in the CEO's payoff because the CEO's compensation partly depends on the firm's performance. Similarly, this dependence on y^t can capture the CEO's ownership of shares in the firm or the future expected payoff associated with the CEO's reputation. After taking the expectation over ε^1 and ε^2 and removing the fixed component of y^t , the period-1 manager's objective function is reduced to

$$U_m = [\chi - d \cdot \mathbf{1}_{\{a^1 \neq \omega^1(\theta^1)\}}] + x \cdot [\chi - d \cdot \mathbf{1}_{\{a^2 \neq \omega^2(\theta^2)\}}]. \quad (1)$$

A manager hired in period 2 aims to maximize $\chi - d \cdot \mathbf{1}_{\{a^2 \neq \omega^2(\theta^2)\}}$.

The sequence in each period is summarized as follows:

Stage 1: In the first period only, shareholders determine the cost or benefit, c , that the board receives in the event that the CEO is removed at the end of $t = 1$.

Stage 2: The manager privately observes her type, θ , and submits a report of her type to the board, $\hat{\theta}^t$.

Stage 3: The board learns $\omega^t(\hat{\theta}^t)$, and sends this information to the CEO. The CEO then takes action a^t .

Stage 4: The board observes the firm's output, $y^t = \theta^t - d \cdot \mathbf{1}_{\{a^t \neq \omega^t(\theta^t)\}} + \varepsilon^t$. When $t = 1$, the board then decides whether or not to replace the CEO. If the CEO is replaced, a new manager arrives with type independently drawn. In the case of $t = 2$, the firm is liquidated and payoffs are realized.

Note that the board can replace the CEO only at Stage 4. The board may already become confident at Stage 2 that the CEO is not worth retaining, but we assume that the

²¹We note that the results are unaffected if we assume that the CEO receives the private benefit χ only in the second period (if retained). We include χ in the first period for consistency.

board cannot immediately dismiss the manager. This assumption corresponds to various administrative difficulties associated with the replacement of a CEO, especially the time devoted to the CEO search. Similarly, the projects under the incumbent CEO's leadership (e.g., acquisitions already in progress) may require her input for a certain period of time. As a result of this assumption, even when the CEO is doomed to be replaced, she still serves as CEO until the end of the period and receives the compensation tied to the firm's performance y^1 . If we relax this assumption on the replacement timing, some of our results would be qualitatively affected.

We employ perfect Bayesian equilibrium as our solution concept. In particular, the board needs to update its belief over the manager's type θ^t once after observing report $\hat{\theta}$, and once again after observing the output y^t . In this way, we can minimize measure-theoretic anomalies associated with conditional probability. (See Appendix A for more technical details.)

3 Optimality of aggressive boards

We solve a model with two manager types as the primary analysis of this paper. The CEO is either the high type θ_H or the low type θ_L , where $\theta_H > \theta_L$. Let π denote the probability of $\theta = \theta_H$. The manager may report either $\hat{\theta} = \theta_H$ or $\hat{\theta} = \theta_L$.

We first characterize a unique equilibrium for each fixed value of the aggressiveness parameter c in Section 3.1. We then determine the optimal level of aggressiveness that shareholders choose to maximize the value of the firm in Section 3.2. Lastly, in Section 3.3, we look into each of the variables to identify economic conditions that make the optimal board aggressive.

3.1 Characterization of equilibrium and the optimal board

Observe that the CEO in the second period has no reason to misreport her type, as there is no replacement decision in the second period. Thus, the CEO's interest is totally aligned with the shareholders' so that she maximizes the firm's value by reporting truthfully and learning the true state $\omega^2(\theta)$. In what follows, we focus on the first period. For ease of exposition, we omit the superscript that indicates period 1.

We primarily focus on equilibria where the CEO with high productivity θ_H reports truthfully to the board. Due to the signaling nature of the model, we must employ an equilibrium selection criterion to preclude an equilibrium where both types are pooled to the low message θ_L . Conversely, it is relatively easy to eliminate the possibility that the manager with θ_H randomizes her report. We postpone the discussion on this issue (until Theorem 1). We

allow the manager with low productivity θ_L to report θ_H with probability $\sigma \in [0, 1]$.

Optimal retention policy

We first consider the optimal retention policy of the board where the replacement cost c is exogenously given. Upon replacement, the board incurs the payoff of c , the cost or subsidy of replacement, and then hires a new manager whose expected productivity is $\mathbb{E}[\theta]$. Hence, the board replaces the CEO when the expected value of θ conditional on the current information (report $\hat{\theta}$ and cash flow y) is below $\mathbb{E}[\theta] + c$. This condition is written as:

$$\mu\theta_H + (1 - \mu)\theta_L < \mathbb{E}[\theta] + c,$$

where μ is the posterior probability of θ_H after observing $\hat{\theta}$ and y . The board keeps the manager when the inequality is reversed.

We classify the values of $\mathbb{E}[\theta] + c$ into several cases. When $\mathbb{E}[\theta] + c > \theta_H$, the board is so hostile that the CEO needs to leave her position regardless of the realized cash flow y . Similarly, if $\mathbb{E}[\theta] + c < \theta_L$, the board is extremely friendly and always retains the CEO. When $\mathbb{E}[\theta] + c$ is either θ_H or θ_L , the problem is subtle: continuously many retention policies are sustainable in this stage because the board is indifferent in keeping the type corresponding to the value $\mathbb{E}[\theta] + c$ (either θ_H or θ_L). We begin with the more straightforward and interesting case of $\mathbb{E}[\theta] + c \in (\theta_L, \theta_H)$.

We observe that the posterior μ is increasing in y (unless $\sigma = 0$, i.e., no mimicry by θ_L). The posterior probability of θ_H is given by:

$$\mu(y; \sigma) = \frac{\pi f(y - \theta_H)}{\pi f(y - \theta_H) + (1 - \pi)\sigma f(y - (\theta_L - d))} = \frac{\pi}{\pi + (1 - \pi)\sigma R(y)}, \quad (2)$$

where $R(y) = f(y - (\theta_L - d))/f(y - \theta_H)$. The likelihood ratio $R(y)$ is decreasing due to the monotone likelihood ratio property. Hence, $\mu(y; \sigma)$ is increasing in y whenever $\sigma > 0$.

The monotone belief implies that the board's retention policy after the good report $\hat{\theta} = \theta_H$ takes the form of a *cutoff rule*. As y increases, the posterior probability improves and eventually the expected value $\mu\theta_H + (1 - \mu)\theta_L$ exceeds the threshold $\mathbb{E}[\theta] + c \in (\theta_L, \theta_H)$. This occurs when μ is equal to

$$\mu^*(c) = \frac{\{\mathbb{E}[\theta] + c\} - \theta_L}{\theta_H - \theta_L} = \pi + \frac{c}{\theta_H - \theta_L}.$$

Due to the monotonicity of μ^* , we can uniquely find a threshold $k \in [-\infty, +\infty]$ such that $\mu(y; \sigma) > \mu^*$ for $y > k$ and $\mu(y; \sigma) < \mu^*(c)$ for $y < k$. That is, the CEO remains in her

position if the observed cash flow y exceeds the *cutoff* k and the board replaces her when $y < k$.

For each value of $\sigma \in (0, 1]$, we have found a unique cutoff $k(\sigma)$ as the best response to the low-type CEO's mixed strategy σ . The best-response function

$$k(\sigma) = R^{-1} \left(\frac{\pi}{1 - \pi} \cdot \frac{1 - \mu^*(c)}{\mu^*(c)} \cdot \sigma^{-1} \right), \quad (3)$$

is continuous and decreasing on $(0, 1]$. Even at $\sigma = 0$, the function $k(\sigma)$ is continuous because $k(\sigma)$ goes to $-\infty$ as $\sigma \rightarrow 0$ and $k = -\infty$ is the optimal cutoff when $\sigma = 0$.

Misreporting

We now analyze the reporting behavior of the CEO with low productivity θ_L given the board employs cutoff k after observing the good report $\hat{\theta} = \theta_H$. If the CEO truthfully reports her low productivity, then the board becomes confident that the manager's type is θ_L and replaces her for sure (i.e., $k = +\infty$).²² On the other hand, after misreporting her type, the CEO may keep her position and gain the private benefit χ at the cost d of productivity loss due to the inaccurate advice from the board. The manager is successful in retention if the outcome $y = \theta_L - d + \varepsilon$ exceeds the cutoff k . Thus, the probability of retention is $1 - F(k + d - \theta_L)$. When the low-ability CEO mixes her report, she must be indifferent between the two choices:

$$\{1 - F(k + d - \theta_L)\} \cdot \chi = d. \quad (4)$$

The left-hand side represents the benefit of misreporting, whereas the right-hand side is the cost, or productivity loss.

By solving the indifference condition (4) in k , we uniquely obtain a cutoff k_0 that makes θ_L indifferent between misreporting and truth-telling:

$$k_0 = \theta_L - d + F^{-1}(1 - d/\chi). \quad (5)$$

If the cutoff k for the good report $\hat{\theta} = \theta_H$ is higher than this threshold cutoff k_0 , the low-type CEO rather prefers to tell the truth even though she will face certain replacement. If $k < k_0$, then θ_L chooses to mimic θ_H to have an acceptable likelihood of retention. We therefore

²²This argument implicitly assumes that the manager with θ_L truthfully reports with positive probability (i.e., $\sigma < 1$). If not, the board may have an arbitrary belief when $\hat{\theta} = \theta_L$. Nevertheless, the following argument on uniqueness still works in the sense that such improvement in belief for $\hat{\theta} = \theta_L$ further reduces the set of possible equilibria.

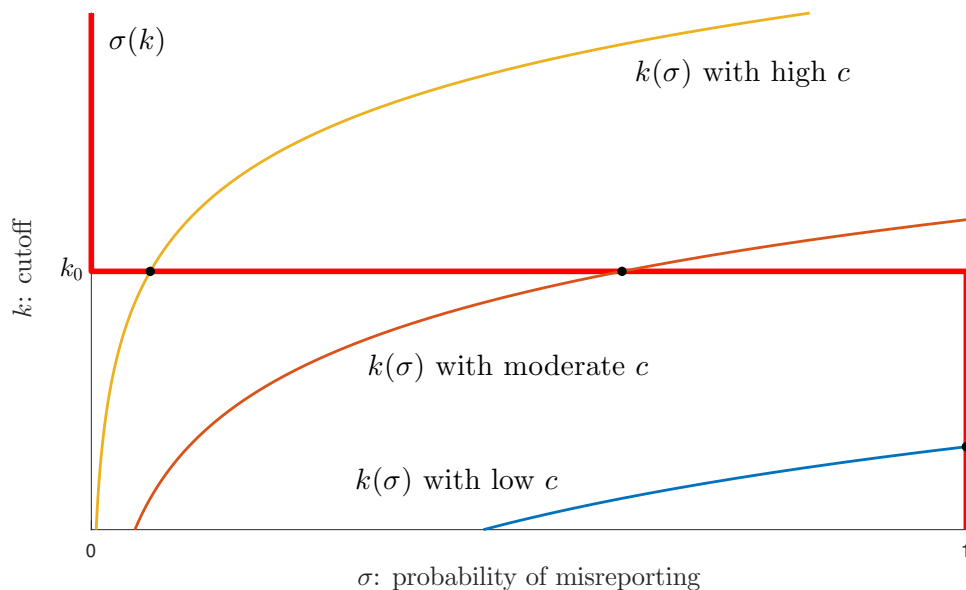


Figure 2: The fixed point problem to find (k^*, σ^*) .

obtain the following best response correspondence for each value of cutoff k for the good report $\hat{\theta} = \theta_H$:

$$\sigma(k) = \begin{cases} \{0\} & \text{if } k > k_0 \\ [0, 1] & \text{if } k = k_0 \\ \{1\} & \text{if } k < k_0. \end{cases} \quad (6)$$

Equilibrium with an exogenous board policy c

An equilibrium is represented by a pair of the cutoff k^* and the mixed strategy σ^* that satisfies $\sigma^* \in \sigma(k^*)$ and $k^* = k(\sigma^*)$. As illustrated in Figure 2, the function $k(\sigma)$ is an increasing function of σ ; an increase in σ decreases μ in equation (2) so that the board must increase k to keep $\mu(k; \theta_H)$ equal to μ^* . Hence, we can find a unique fixed point.

Before claiming the game has a unique equilibrium, we need to consider the possibility of non-plausible equilibria where both types report $\hat{\theta} = \theta_L$. Such equilibria do not survive the D1 criterion. We consider the model with exogenous c as a signaling game by interpreting the report $\hat{\theta}$ as the message from the sender (the CEO) and the retention policy as the action from the receiver (the board). We apply the D1 criterion to such a signaling game (see Appendix A for more details). In Appendix B, we show that, when the message θ_H

is an out-of-equilibrium message, it is type θ_H that benefits the most from employing that message because of d , the disadvantage from misreporting.

Theorem 1. *Consider the two-type model with exogenously given c .*

- (i) *When $c \in (\theta_L - \mathbb{E}[\theta], \theta_H - \mathbb{E}[\theta])$, an equilibrium that survives the D1 criterion uniquely exists. In the unique equilibrium,*
- *the high-type CEO sends a truthful report for sure, but the low-type CEO sends report θ_H with probability σ^* ;*
 - *after report θ_H , the board employs a cutoff k^* ; and*
 - *after report θ_L , the board assigns probability 1 on type θ_L and replaces the CEO for sure.*

The parameters σ^ and k^* are characterized by*

- (a) $\mu(k^*; 1) = \mu^*(c)$ and $\sigma^* = 1$ when $\mu^*(c) \leq \mu(k_0; 1)$; and
 (b) $k^* = k_0$ and $\mu(k_0; \sigma^*) = \mu^*(c)$ when $\mu^*(c) \geq \mu(k_0; 1)$.

Furthermore, there exists $\hat{c} \in (\theta_L - \mathbb{E}[\theta], \theta_H - \mathbb{E}[\theta])$ such that $c > \hat{c}$ implies $\mu^(c) > \mu(k_0; 1)$ and $c < \hat{c}$ implies $\mu^*(c) < \mu(k_0; 1)$.*

- (ii) *When $\mathbb{E}[\theta] + c > \theta_H$, the game has a unique equilibrium where the board always replaces the CEO regardless of her report and output.*
 (iii) *When $\mathbb{E}[\theta] + c < \theta_L$, the game has a unique equilibrium where the board always keeps the CEO regardless of her report and output.*

Theorem 1 provides existence and uniqueness under the D1 criterion, and also specifies that there exists a level \hat{c} in the relevant domain $(\theta_L - \mathbb{E}[\theta], \theta_H - \mathbb{E}[\theta])$ which induces partial separation of the high type. As formally shown in Theorem 1 and as illustrated by Figure 2, there are two possible classes of D1 equilibria. The first type of equilibrium appears on the vertical area to the right in Figure 2: the low-type manager always misreports (i.e., full pooling where $\sigma = 1$) and the cutoff k is implicitly determined by $\mu(k; 1) = \mu^*$. The second type, partial separation, is represented by the dots in the horizontal area. Contrary to the first, in this type of equilibrium, the cutoff is unchangeably set to k_0 whereas σ is implicitly given by $\mu(k_0; \sigma) = \mu^*$; here, $k = k_0$ is plugged into the condition $\mu(k; \sigma) = \mu^*$.

Theorem 1 and Figure 2 also show that these two equilibria continuously arise. As we see from Figure 2, the board's inclination for removal, c , determines the cutoff level for replacement and the low type's frequency of misreporting. When c is sufficiently small (i.e., smaller than \hat{c}), the board is sufficiently lenient so that the equilibrium of the first kind discussed above (i.e., pooling equilibrium) emerges. As c increases, while the probability σ

of misreporting is kept equal to 1, the equilibrium cutoff increases and eventually reaches k_0 —the right-most corner of the horizontal line in Figure 2—when c touches the threshold value \hat{c} . After that, the equilibrium moves along the horizontal area and reduces the probability σ of misreporting while keeping the value of the equilibrium cutoff k equal to k_0 . As $\mathbb{E}[\theta] + c$ approaches θ_H , the misreporting probability σ eventually converges to 0. This argument is formally shown in Theorem 1.

We briefly discuss what happens when $\mathbb{E}[\theta] + c > \theta_H$ and when $\mathbb{E}[\theta] + c < \theta_H$. When the board is extremely hostile (Theorem 1 (ii)), the manager has no chance to remain in the firm and thus attempts to maximize the value of the firm during her tenure by reporting truthfully. Similarly, if the board is extremely friendly (Theorem 1 (iii)), the manager faces no threat of replacement and thus her objective is again totally aligned with that of the shareholders.

Multiple equilibria emerge in the knife-edge cases of $\mathbb{E}[\theta] + c = \theta_H$ and $\mathbb{E}[\theta] + c = \theta_L$. Some of the equilibria are pathological and make the shareholders' payoff discontinuous with respect to c on $[\theta_L - \mathbb{E}[\theta], \theta_H - \mathbb{E}[\theta]]$, although they do not survive after we introduce small perturbations to the model.²³ To avoid complications associated with multiple equilibria, we allow equilibria that are continuously connected to the equilibria described in Theorem 1.²⁴ More specifically, we allow only the following equilibria:

- (i) When $\mathbb{E}[\theta] + c = \theta_H$, the board uses the cutoff k_0 when the CEO reports θ_H and replaces her when θ_L . The CEO always reports her true type.
- (ii) When $\mathbb{E}[\theta] + c = \theta_L$, the board keeps the CEO when she reports θ_H and replaces her when she reports θ_L . The CEO always reports θ_H .

3.2 Optimal board policy

We now investigate the degree of board hostility or friendliness, c , that the shareholders endogenously determine. By changing c , shareholders are able to manipulate the equilibrium values of the mimicry propensity σ and cutoff k . As shown in Figure 2, when c is low enough, these values start from the right vertical line segment representing $\sigma = 1$ and $k \leq k_0$. That is, the board is lenient enough that the low type never communicates truthfully. As c increases, the cutoff k increases and eventually hits $k = k_0$. After that, the cutoff becomes constant $k = k_0$ but the misreporting probability σ begins to decline.²⁵ The equilibrium

²³For example, we can replace the atomic type θ_i with a uniform distribution on $(\theta_i - \Delta, \theta_i + \Delta)$ with small Δ . The equilibrium multiplicity disappears as we see in Internet Appendix C.

²⁴That is, we eliminate equilibria that are discontinuous from neighboring equilibria. In other words, we impose some sort of lower hemi-continuity on equilibria in the two knife-edge cases.

²⁵While the cutoff k is constant with increases in c , the mimicry probability decreases in order to maintain the heightened retention standard by the board.

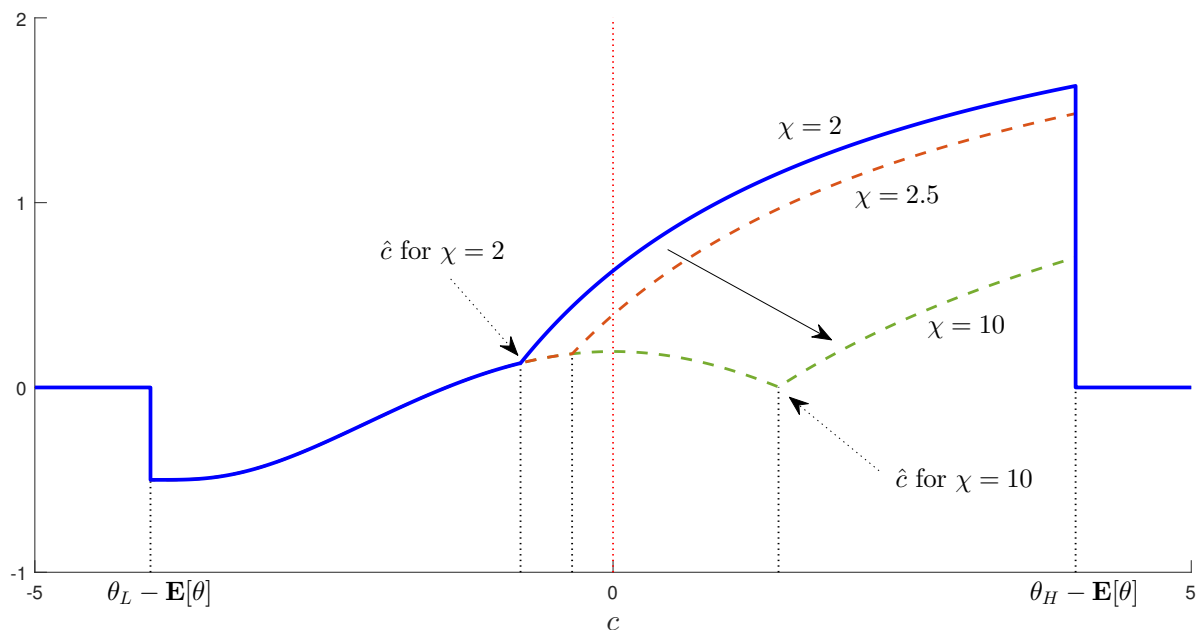


Figure 3: The normalized objective function of the shareholders when $\theta_H = 9$, $\theta_L = 1$, $\pi = 1/2$, $d = 1$, and $\varepsilon \sim N(0, 10)$.

moves leftward and eventually reaches $\sigma = 0$ when c arrives at the maximum aggressiveness $c = \theta_H - \mathbb{E}[\theta]$.

We ultimately observe that only two values of c survive as candidates of the optimal choice: $c = 0$ (*neutral* board) and $c = \theta_H - \mathbb{E}[\theta]$ (*maximally aggressive* board). As Figure 3 illustrates, the maximally aggressive board $c = \theta_H - \mathbb{E}[\theta]$ arises as the primary candidate of the solution. Consider \hat{c} as in Theorem 1. The maximum aggressiveness $c = \theta_H - \mathbb{E}[\theta]$ dominates any other $c \in (\hat{c}, \theta_H - \mathbb{E}[\theta])$, as they all induce the same cutoff k_0 , but the maximally aggressive board minimizes the misreporting probability σ to 0 unlike any other value of c .

When the private benefit χ is large (see the case of $\chi = 10$), the objective function of the shareholders is no longer monotonic and the neutral board $c = 0$ emerges as another peak. As illustrated in Figure 4, the neutral board, $c = 0$, tends to be optimal when χ is large. Intuitively, the neutral board emerges as a local optimum because this board calculates the least distortive equilibrium cutoff k^* . Hence, we ultimately see that two potential peaks emerge in the shareholders' payoff. The first is at $c = \hat{c}$ and the second at $c = \theta_H - \mathbb{E}(\theta)$.

We formalize the above arguments. We first formulate the objective function of the shareholders. Recall that the shareholders' objective is to maximize the sum of cash flows

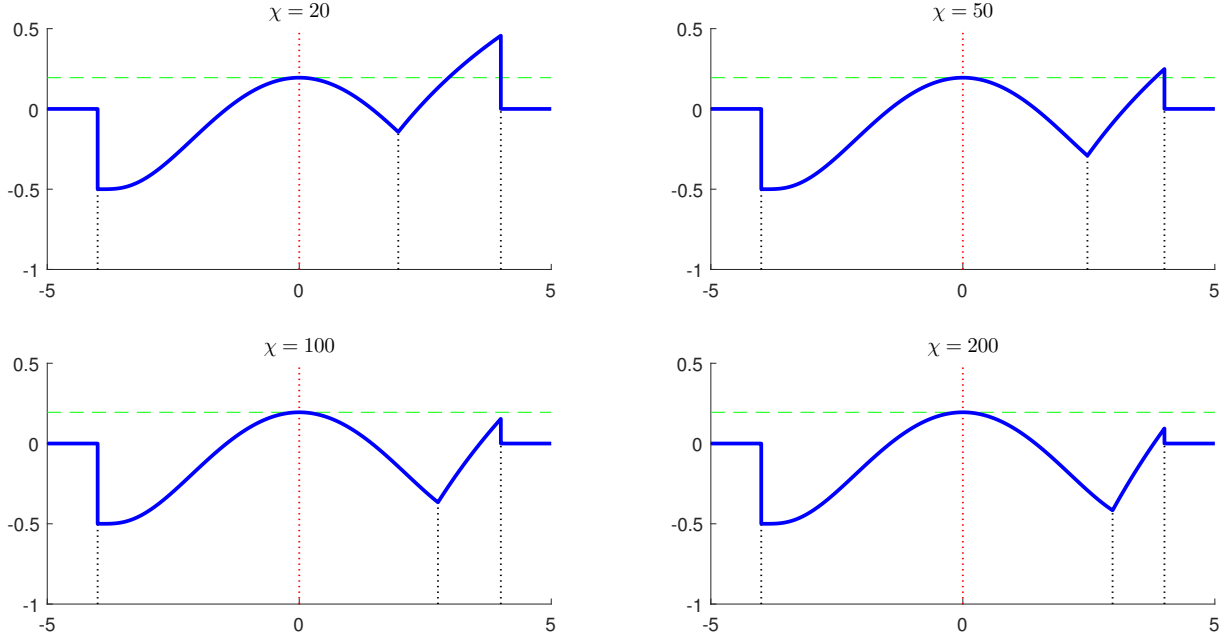


Figure 4: The normalized objective function of the shareholders when $\theta_H = 9$, $\theta_L = 1$, $\pi = 1/2$, $d = 1$, and $\varepsilon \sim N(0, 10)$.

in the two periods. Given σ , the expected value of the output in period 1 is

$$\tilde{V}_1(\sigma) = \mathbb{E}[\theta] - (1 - \pi)\sigma d.$$

The expected value of period-2 output depends on how likely the manager of each type remains in the firm. The probability of retention is $1 - F(k - \theta_H)$ ($= F(\theta_H - k)$) for type θ_H , $F(\theta_L - k - d)$ for type θ_L who misreports, and 0 for type θ_L who reports truthfully. Since the expected productivity is $\mathbb{E}[\theta]$ after replacement, the expected value of the output in period 2 is:

$$\begin{aligned} \tilde{V}_2(\sigma, k) &= \mathbb{E}[\theta] + \pi(\theta_H - \mathbb{E}[\theta])F(\theta_H - k) + \sigma \cdot (1 - \pi)(\theta_L - \mathbb{E}[\theta])F(\theta_L - k - d) \\ &= \mathbb{E}[\theta] + \Delta\theta \cdot \pi(1 - \pi) [F(\theta_H - k) - \sigma F(\theta_L - k - d)], \end{aligned}$$

where $\Delta\theta = \theta_H - \theta_L$. It is noteworthy that we can combine the terms for the two types despite the different weights (π and $1 - \pi$) because $\theta_H - \mathbb{E}[\theta] = (1 - \pi) \cdot \Delta\theta$ is proportional to $1 - \pi$ and $\theta_L - \mathbb{E}[\theta] = \pi \cdot \Delta\theta$ is proportional to π . By combining these two values, we obtain

the following objective function (after subtracting the constant $\mathbb{E}[\theta]$ twice for normalization):

$$V(\sigma, k) = (1 - \pi) \left\{ \Delta\theta \cdot \pi [F(\theta_H - k) - \sigma F(\theta_L - k - d)] - d\sigma \right\}.$$

The shareholders' objective is to maximize this function V .

Now, we investigate the optimal value of c in the interval $[\hat{c}, \theta_H - \mathbb{E}[\theta]]$. In this case, the cutoff k is a constant ($k^* = k_0$ by Theorem 1) but the misreporting rate σ decreases as c increases, as illustrated in Figure 2 (the red horizontal line). We refer to this reduction of misreporting probability as the *disciplinary effect*, and to this interval as the *disciplined region*. The objective function $V(\sigma, k)$ is decreasing in σ , and σ shrinks to 0 as $c \rightarrow \theta_H - \mathbb{E}[\theta]$. Therefore, the maximally aggressive c is optimal for shareholders in the disciplined region $[\hat{c}, \theta_H - \mathbb{E}[\theta]]$, as stated in the lemma below. In other words, the value of the firm keeps increasing, as c increases, due to the disciplinary effect. Note that the equilibrium cutoff level is maintained at $k^* = k_0$ in the limit $c = \theta_H - \mathbb{E}[\theta]$, even though this c signifies maximum aggressiveness.²⁶

Lemma 1. *Assume that the board employs the cutoff k_0 when $\mathbb{E}[\theta] + c = \theta_H$. On the interval $[\hat{c}, \theta_H - \mathbb{E}[\theta]]$, the optimal value of c for the shareholders is $c = \theta_H - \mathbb{E}[\theta]$.*

In contrast, we refer to the region $[\theta_L - \mathbb{E}[\theta], \hat{c}]$ as the *incurrible region*, as changes in c do not affect the value of σ , but rather affect the value of the cutoff k . In this region, through controlling the board, shareholders wish to achieve the optimal value of k under the constraint that $\sigma = 1$ is unchangeable. As we see below, the solution is simple: the board should share the same interest with shareholders (i.e., $c = 0$) whenever possible. In this way, the board never makes a biased decision—such a bias is useful in the disciplined region, but not in the incurrible region—and thus the equilibrium cutoff k is optimal for shareholders. In other words, by choosing a value of c other than $c = 0$, shareholders end up with distorting the retention decision of the board and reducing the payoff for themselves. We refer to this effect as the *distortion effect*.

To see this point more explicitly, consider the problem of maximizing shareholders' value $V(\sigma, k)$ given $\sigma = 1$. The first-order condition

$$\pi(\theta_H - \theta_L) \{f(k + d - \theta_L) - f(k - \theta_H)\} = 0,$$

is equivalent to $R(k) \equiv f(k - (\theta_L - d))/f(k - \theta_H) = 1$. When $c = 0$, $R(k)$ is indeed equal

²⁶As stated at the end of Section 3.1, we consider the equilibrium that is consistent with equilibria before the limit. These equilibria employ $k^* = k_0$ as the equilibrium cutoff, and thus the limit equilibrium also employs this cutoff level.

to 1 in equilibrium because the unique solution of the indifference condition

$$\pi = \mu^* = \mu(y; 1) = \frac{\pi}{\pi + (1 - \pi)R(k)},$$

is $R(k) = 1$. Therefore, $c = 0$ is the optimal choice if the neutral board is included in the incorrigible region (i.e., $0 \leq \hat{c}$).

The next lemma describes the optimal value of c in the incorrigible region. The optimal value of c minimizes the board's bias and thus the distortion effect.

Lemma 2. *On the interval $(\theta_L - \mathbb{E}[\theta], \hat{c}]$, the optimal value of c for the shareholders is $c = 0$ if $0 \leq \hat{c}$; otherwise, $c = \hat{c}$.*

Lemma 2 states that, in the incorrigible region, shareholders set the board to be neutral when $\hat{c} \geq 0$, and set the board to be friendly when $\hat{c} \leq 0$ by setting $c = \hat{c}$ in the latter case. However, we ultimately show (in the next theorem) that the friendly board which occurs when $\hat{c} \leq 0$ is dominated by an aggressive board after we consider the whole interval $(\theta_L - \mathbb{E}[\theta], \theta_H - \mathbb{E}[\theta])$. This occurs because, even though $c = \hat{c}$ is optimal within the incorrigible region, setting $c = \hat{c}$ is the *worst* choice in the disciplined region. In contrast, in the case of $\hat{c} > 0$, the payoff function V of shareholders has two peaks, as illustrated in Figure 4. Hence, on the interval $(\theta_L - \mathbb{E}[\theta], \theta_H - \mathbb{E}[\theta])$, two choices for the shareholders' optimal board policy emerge: either neutral ($c = 0$) or aggressive ($c > 0$).

We now characterize the necessary and sufficient condition for the optimality of the maximally aggressive board. In the statement below, we can interpret the variable $m = F^{-1}(1 - d/\chi)$ as a measure of the CEO's *intrinsic misreporting incentive*: as $\chi \rightarrow \infty$, the value of m goes to infinity, whereas m goes to $-\infty$ as the cost of miscommunication, d , approaches the value of the private benefit, χ . The condition $m = m_*$ provides a threshold in terms of the intrinsic incentive of misreporting for the CEO of low ability.

Theorem 2. *Let $m = F^{-1}(1 - d/\chi)$ and $m_* = (\Delta\theta + d)/2$.*

- (i) *When $m \leq m_*$, the payoff of the shareholders is increasing in c on $(\theta_L - \mathbb{E}[\theta], \theta_H - \mathbb{E}[\theta])$. Consequently, the maximally aggressive board is optimal.*
- (ii) *When $m > m_*$, the payoff of the shareholders has two peaks on $(\theta_L - \mathbb{E}[\theta], \theta_H - \mathbb{E}[\theta])$: $c = 0$ and $c = \theta_H - \mathbb{E}[\theta]$. The aggressive board, $c = \theta_H - \mathbb{E}[\theta]$, is optimal if and only if*

$$F(2m_* - m) + \frac{d}{\pi \cdot \Delta\theta} \geq 2F(m_*) - 1. \quad (7)$$

It is the unique optimum when the inequality is strict.

Figure 3 is helpful in understanding part (i) of Theorem 2. Let us start from $c = \theta_L - \mathbb{E}[\theta]$ and gradually increase the value of c . The value of the shareholders' objective function initially increases because c approaches the neutral level $c = 0$ in the incorrigible region. If $\hat{c} \leq 0$, the value of c reaches \hat{c} before the objective function starts to decline. Now the value of c enters the disciplined region $[\hat{c}, \theta_H - \mathbb{E}[\theta]]$ and the objective function continues to increase. Thus, as long as $\hat{c} \leq 0$, the objective function is increasing on $[\theta_L - \mathbb{E}[\theta], \theta_H - \mathbb{E}[\theta]]$. We show that the condition $\hat{c} \leq 0$ is equivalent to $m \leq m_*$ in Appendix B.

In part (ii) of Theorem 2, we obtain condition (7) by simply comparing the two cases. When $\hat{c} > 0$, the equilibrium with $c = 0$ is given by $\sigma = 1$ and $R(k) = 1$. We can explicitly solve the equation $R(k) = 1$. With its unique solution $k^{**} = (\theta_H + \theta_L - d)/2$, we can explicitly calculate the value of $V(\sigma, k)$ when $c = 0$:

$$\begin{aligned} V_N = V(1, k^{**}) &= \Delta\theta \cdot \pi(1 - \pi) \left[F(m_*) - F(-m_*) - \frac{d}{\pi \cdot \Delta\theta} \right] \\ &= \Delta\theta \cdot \pi(1 - \pi) \left[2F(m_*) - 1 - \frac{d}{\pi \cdot \Delta\theta} \right]. \end{aligned}$$

When the board is maximally aggressive (i.e., $\mathbb{E}[\theta] + c = \theta_H$), $\sigma = 0$ and the shareholders' payoff is given as:

$$V_A = V(0, k_0) = \Delta\theta \cdot \pi(1 - \pi)F(\theta_H - k_0) = \Delta\theta \cdot \pi(1 - \pi)F(2m_* - m).$$

Condition (7) is determined by comparing V_A and V_N .

This result is in stark contrast to several theoretical studies which have found that management-friendly boards can facilitate communication between the CEO and the board. The prior theoretical literature that examines strategic communication between the CEO and the board generally finds that greater alignment in preferences between the two parties enhances communication (Adams and Ferreira (2007), Harris and Raviv (2008), Baldenius et al. (2014), Chakraborty and Yilmaz (2017)). In contrast, in this model we find that greater *misalignment* in preferences between the board and CEO (i.e., with regard to the replacement decision) facilitates effective communication (the disciplinary effect).²⁷ By considering the interrelationship between advising and replacement, we find that aggressive boards—boards with greater misalignment with CEOs—are often optimal for shareholders to improve communication.

One of the strengths of our parsimonious model is that we can more directly observe the two disparate, countervailing effects of aggressive boards. Moreover, as shown above, we

²⁷See Section 6.1 for further discussions on the effect of aggressiveness on communication.

observe these two effects distinctly separated in two regions. This allows us to cleanly observe how changes in the shareholders' choice of c affect the board's cutoff k and the manager's reporting strategy σ . The first effect from a policy of aggressive boards is on the probability of misreporting. We find that the low type's equilibrium probability of misreporting, σ , decreases in the disciplined region as c increases. Intuitively, a stricter retention policy disincentivizes low-quality CEOs to engage in value-reducing mimicry in order to receive personal benefits from continuation. As the second-period benefit from retaining power is reduced, CEOs become more focused on first-period advisement to increase firm value. Consequently, the low-type CEO's payoff is collapsed to the first period, thus aligning the CEO's incentives with shareholders to maximize firm value. In sum, misalignment between the board and the CEO in the retention decision disciplines the CEO and induces honesty in the CEO's report.

The second effect is the costly distortion in the retention policy. By making the board more aggressive, a higher hurdle for retention is adopted. Although the higher cutoff disciplines the manager with low productivity, excessive replacement decisions can be detrimental for firm value as talented CEOs are mistakenly removed. We characterize conditions under which the first effect dominates and the aggressive board is optimal. In the case where the second effect dominates, the results show that a neutral board is optimal. Perhaps surprisingly, Theorem 2 shows that friendly boards do not emerge along the equilibrium path.

We note that the absence of friendliness arises from the interaction of advising and replacement. Due to the value-increasing nature of advising, the CEO and the board are partially aligned in preferences through the benefit from informative communication. In this regard, friendly and aggressive boards share a similar purpose—both types of boards weaken the tie between firm performance and CEO turnover, thus enhancing the CEO's incentive for honest communication. However, the aggressive board has an additional advantage in that it can replace low-type CEOs, whereas a board friendly enough to induce truthful communication keeps them in the second period. This benefit of greater CEO separation from an aggressive board improves future firm value.

3.3 When the optimal board is aggressive

In this section, we investigate the variables of this model in light of Theorem 2 to identify economic conditions that drive the optimal board to be aggressive. Before proceeding, we additionally assume that the noise term ε is normally distributed with mean 0 and variance σ_ε^2 . This specification allows us to make a formal statement on the variance of ε . Also, the productivity difference $\Delta\theta$ can naturally be interpreted as a variance parameter for the

productivity type θ (with π fixed), because the variance of θ is $\pi(1-\pi)(\Delta\theta)^2$. We summarize the results we discuss in this section in the following corollary of Theorem 2.

Corollary 1. *An aggressive board is optimal for shareholders when one of the following holds:*

- (i) *The benefit from truthful communication, d , is sufficiently large.*²⁸
- (ii) *The concentration of low-ability CEOs is sufficiently high (i.e., the probability of the high type, π , is low enough).*
- (iii) *The variance of the noise term ε is sufficiently high.*²⁹
- (iv) *With fixed π , the variance of CEO ability θ is sufficiently low (i.e., $\Delta\theta$ is low enough).*
- (v) *The private benefit χ is sufficiently low.*

In what follows, we discuss the economic forces that drive each of the statements in Corollary 1. First, we see in Corollary 1 (i) that shareholders choose an aggressive board when the value of communication, d , is sufficiently large (note that $m_* - m$ is increasing in d and goes to ∞ as $d \rightarrow \chi$). When d is close enough to χ , the CEO is discouraged from misreporting and thus the cutoff k_0 , with which the CEO is indifferent between truth-telling and misreporting, decreases. As the indifference cutoff k_0 decreases, it eventually goes below the cutoff *that is optimal for the shareholders*. Because the board's replacement policy becomes unacceptably lenient, the shareholders rather induce the board to be more aggressive by raising c .

Second, the probability of the high type, π , has a negative effect on board aggressiveness, as the right-hand side of equation (7) is increasing in π . Intuitively, when the probability that the CEO is of type θ_L is low, so is the ex ante likelihood of misreporting. Shareholders then focus on avoiding distortionary retention policies and eventually choose a neutral board when π is large enough. Put differently, the disciplinary effect from aggression—truthful reporting from low types—dissipates as π increases. As π approaches 0, the left-hand side of condition (7) unboundedly increases and condition (7) is satisfied. Hence, shareholders prefer the board to be more aggressive when there is a high probability mass of low types.

Third, shareholders prefer a more aggressive board as the variance of noise increases. In this case, the support of ε expands and the low-type manager has a higher chance of successfully mimicking the high type in her observed output. Likewise, there is less room for the high type to meet the cutoff with certainty. As a result, the cutoff-based retention policy becomes less effective and the choice of cutoff k becomes less important. In other words, the increase in the noise dilutes the distortion effect of aggressiveness, while the disciplinary

²⁸Once d exceeds χ , both types report truthfully regardless of the board's retention policy.

²⁹For this comparative static, we have assumed that ε is normally distributed.

effect, represented by \tilde{V}_1 , is unchanged. With the disciplinary effect intact but the distortion effect weakened, shareholders face an incentive to make the board *more* aggressive in response to an increase in the variance of ε .

Next, shareholders optimally choose the aggressive board when the difference $\Delta\theta = \theta_H - \theta_L$ is small enough to satisfy inequality (7). In an extreme situation where θ_H and θ_L are almost identical, the choice of cutoff k is mostly unimportant to the shareholders because they can hire a similarly talented CEO even after wrongly replacing a high-type one. In other words, the distortion effect disappears as $\Delta\theta$ shrinks to 0, while the disciplinary effect is intact. Similarly, as the dispersion in types increases, the shareholders face an increased risk in replacing a highly talented CEO. The increase in variance amplifies the disutility from the distortion effect of a high cutoff k , and leads shareholders to prefer a less aggressive board.

Lastly, the aggressive board is optimal when the private benefit of control χ is sufficiently small. Intuitively, a decrease in χ reduces the low type CEO's incentive to misreport while keeping the shareholders' preferences over the board's replacement cutoff unchanged. Consequently, as occurs with the value loss d , the cutoff k_0 that makes the low-type CEO indifferent eventually becomes lower than the standard the shareholders would set, resulting in shareholders setting a more aggressive board.

Theorem 2 and Corollary 1 have a number of implications. We present a thorough discussion of the empirical predictions which emerge from our analysis (and their corresponding relation to the extant empirical literature) in Section 5.

4 Continuous types

In this section, we examine the case of continuously many types, $\Theta = (\underline{\theta}, \bar{\theta})$. (We have studied discrete types $\Theta = \{\theta_H, \theta_L\}$ in Section 3.) We summarize the main results for continuous types. More detailed explanations and additional results are presented in Internet Appendix C.

As in the two-type setting, the board aims to keep high types and replace lower types. Specifically, the threshold is $\theta_1 = \mathbb{E}[\theta] + c$. In turn, CEOs of type $\theta > \theta_1$ report truthfully in equilibrium, while types below θ_1 need to inflate the report in order to have a chance at retention. However, not all types below θ_1 find misreporting optimal. CEOs with very low types often face an insufficient chance of retention such that the potential gain from mimicry is outweighed by the benefit of truthful communication, d . The lower threshold type θ_2 ,

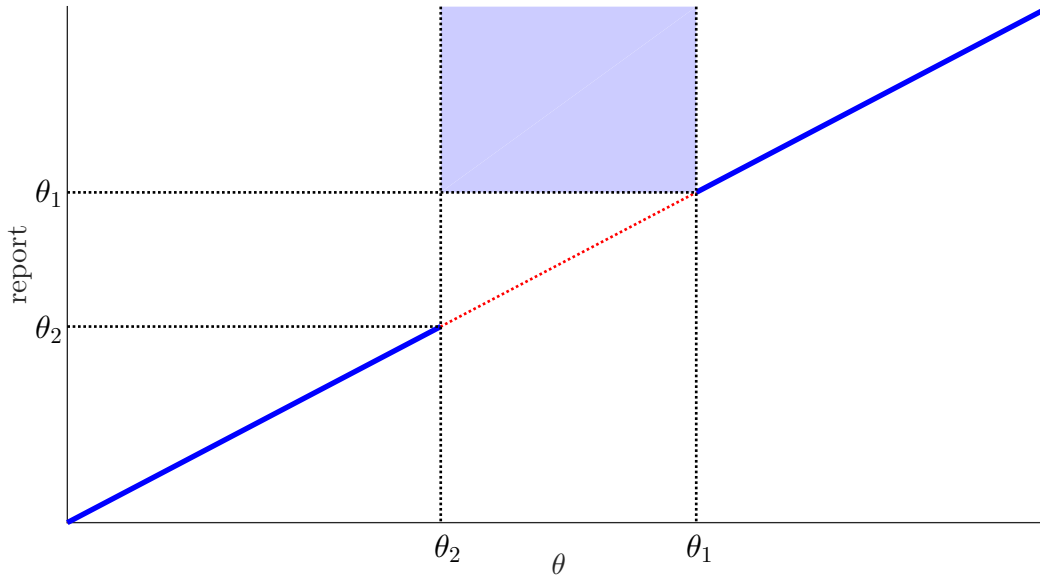


Figure 5: The manager's reporting strategy. The manager with type $\theta \in (\theta_2, \theta_1)$ uses a report in the shaded area.

below which types report truthfully, is uniquely determined by the indifference condition

$$\{1 - F(k_* + d - \theta_2)\} \chi = d,$$

where k_* is the equilibrium cutoff specified below by equation (8). Therefore, the CEO chooses to inflate the message only when the type θ lies between the two thresholds, θ_2 and θ_1 . As illustrated in Figure 5, this strategy is non-monotonic in two senses: the reporting strategy is not always increasing; and the truth-telling region $(\theta, \theta_2) \cup (\theta_1, \bar{\theta})$ has a gap in the middle.

We now turn to the board's decision. Interestingly, we find that the board employs a uniform cutoff k_* for all messages above the standard, $\hat{\theta} \geq \theta_1$ in equilibrium. The board cannot set two different standards k', k'' for two different messages $\hat{\theta}', \hat{\theta}'' \geq \theta_1$, because the message with the lower standard attracts more misreports, as mimicking types will avoid the higher cutoff for retention. At optimum, the uniform cutoff k_* is given by

$$\int_{\theta_1}^{\bar{\theta}} (\theta - \theta_1) f(k_* - \theta) g(\theta) d\theta = \int_{\theta_2}^{\theta_1} (\theta_1 - \theta) f(k_* + d - \theta) g(\theta) d\theta, \quad (8)$$

where $g(\theta)$ is the probability density function for type θ . The left-hand side represents the aggregate positive effect from keeping the current manager across all types above θ_1 , whereas the right-hand side corresponds to the misreporting types, $\theta \in (\theta_2, \theta_1)$. The board finds k_* optimal when these two effects are balanced.

The theorem below summarizes the equilibrium of the continuous model under exogenous $c \in [\underline{\theta} - \mathbb{E}(\theta), \bar{\theta} - \mathbb{E}(\theta)]$.

Theorem 3. *Any equilibrium that survives the D1 criterion satisfies the following properties in the first period:*

- (i) *The manager truthfully reports her type if $\theta \notin [\theta_2, \theta_1]$. The manager with type $\theta \in (\theta_2, \theta_1)$ chooses some report above θ_1 .*
- (ii) *The board replaces the manager when the manager reports $\hat{\theta} < \theta_1$ or the output y is less than k_* .³⁰*

We find that board aggressiveness continues to encourage truthful communication in this setting with continuous types. As the board becomes more aggressive, the equilibrium k_* cutoff increases and θ_2 —the lower end of the mimicry interval (θ_2, θ_1) —also increases. However, the higher end θ_1 also increases; that is, intermediate types $\theta \in (\mathbb{E}[\theta], \theta_1)$ are induced to misreport and forgo the informational gain from truthful communication, an additional cost to aggressiveness. Shareholders consequently must take into consideration this additional cost when determining the optimal board policy.

To analytically determine the shareholders' optimal board policy, we employ additional assumptions to deal with the complexity of the model with a continuous type space. Specifically, we assume that type θ and noise ε are uniformly distributed on supports $[\underline{\theta}, \bar{\theta}]$ and $[-q, q]$, respectively. We define $\Delta = \bar{\theta} - \underline{\theta}$. We also assume the regularity conditions stated in Internet Appendix C. This setting with the simple distributional assumptions enable us to obtain the following analytic solution.³¹

Theorem 4. *It is optimal for shareholders to choose an aggressive board with $c \in (0, \Delta/2)$. The optimal value of c is uniquely given by*

$$c_A^* = \frac{1}{8} \left\{ d \left(1 + \frac{8q}{\chi} \right) + 2\Delta - \sqrt{D} \right\},$$

where $D = d^2(1 + 8q/\chi)^2 + 4d\Delta - 32dq + 4\Delta^2 > 0$.

³⁰The statement of this theorem is simplified from Theorem 5. See Internet Appendix C for more results.

³¹As in the baseline setting, the shareholders' payoff can be divided into the two periods. In the first period, shareholders receive the payoff $V_1 = -d\{G(\theta_1) - G(\theta_2)\}$ plus $\mathbb{E}[\theta]$. Here, the difference $G(\theta_1) - G(\theta_2) = \Pr\{\theta \in [\theta_2, \theta_1]\}$ is the probability of misreporting. The CEO in period 2 never misreports her type, but the board's retention policy in the first period affects the expected value of θ in the second period. The expected second-period value is $V_2 = \int_{\theta_1}^{\bar{\theta}} (\theta - \mathbb{E}[\theta])F(\theta - k)g(\theta) d\theta + \int_{\theta_2}^{\theta_1} (\theta - \mathbb{E}[\theta])F(\theta - k - d)g(\theta) d\theta$ plus $\mathbb{E}[\theta]$. The shareholders maximize the sum $V = V_1 + V_2$ by controlling c . We analytically calculate the values of V_1 and V_2 to determine the optimal policy c .

In contrast to the two-type case, the optimal policy is not maximal aggression. We find that a moderately aggressive board is optimal since the shareholders' disutility from the distorted retention decision k_* eventually exceeds the benefit of the disciplinary effect as the degree of aggressiveness, c , increases.

The explicit solution c_A^* allows for comparative statics analysis. Note that the variables q and Δ are interchangeable with the variances of noise ε and type θ , respectively, in the comparative statics below, because $\text{Var}(\varepsilon) = q^2/3$ and $\text{Var}(\theta) = \Delta^2/12$.

Proposition 1. *The optimal aggressiveness c_A^* is increasing in the cost of miscommunication, d , and in the variance of noise ε , and decreasing in the variance of type θ .*

These comparative statics results are consistent with those presented in Corollary 1 of the baseline setting. An increase in d , the value loss from uninformative communication, results in shareholders setting a more aggressive board to elicit greater truthful reporting, and thus informative communication, from the CEO. Likewise, an increase in the variance of noise or in the variance of CEO type affects the shareholders' payoff in a similar way as in the baseline setting. A more noisy performance measure weakens the distortion effect of the cutoff k_* and drives shareholders to prefer a more aggressive board. In contrast, with a greater variance in θ , shareholders are more likely to replace a high-type CEO, leading the optimal policy to be a less aggressive board.

In Internet Appendix C, we additionally present a number of numerical simulations under different distributions of θ that support the optimality of an aggressive board. We also find that χ negatively affects the optimal aggressiveness when $\Delta = \bar{\theta} - \underline{\theta}$ is reasonably high.

5 Empirical implications

In this section, we discuss connections to extant empirical work and present key empirical predictions that emerge from the analysis. Several of these predictions have not yet been explored in the empirical literature. Our analysis may therefore help to guide future investigation.

Our results imply that shareholders often find optimal a board that aggressively terminates the employment of CEOs. The empirical literature has shown that CEOs who are able to continue their employment tend to exhibit greater performance over CEOs who are terminated early (e.g., Dikolli et al. (2014)). Board aggressiveness *enhances* this tendency whereby CEOs who preserve their position are more likely to be adept relative to those who are removed, because an aggressive board is more effective in rooting out weak CEOs and only retains CEOs with strong performance. This argument implies, in terms of CEO tenure

length, that the gap in performance between longer-tenured CEOs and ones with shorter tenure should be *increasing* in the aggressiveness of the board. Prediction 1 below claims that we should observe variation in this performance gap between longer- and shorter-tenured CEOs depending on board aggressiveness.

Prediction 1. *Consider the performance gap between CEOs with longer tenure and CEOs with shorter tenure. This gap is increasing and more pronounced among firms or industries with more aggressive boards.*

In practice, an aggressive board can be potentially implemented in a number of ways. Shareholders, with wide discretion over board member compensation, can reward board members highly only in periods of very strong firm performance to induce an aggressive CEO replacement practice. Shareholders can also determine the composition and structure of the board. Weisbach (1988), Dahya et al. (2002), and Huson et al. (2004) document that a greater presence of outside directors is positively associated with CEO turnover, while Guo and Masulis (2015) similarly show that board independence and fully independent board nominating committees are linked to greater CEO turnover. Relatedly, shareholders can enhance aggressiveness and increase CEO turnover by adopting a non-staggered (or declassified) board (Faleye (2007)) or by allowing inside directors the possibility of serving as CEO in the event of CEO dismissal (Mobbs (2013)). In addition, shareholders can appoint directors who have been previously involved in a forced CEO turnover (Ellis et al. (2016)) or who hold fewer other directorships (Fich and Shivdasani (2012)). Shareholders can also require regular rotation of board members to reduce board entrenchment (Huang and Hilary (2018)). All of these practices are linked to greater CEO turnover. In terms of firm structure and policies, with fewer obstacles to remove CEOs, boards should be more keen on exercising their replacement authority. In particular, with lower managerial entrenchment, such as fewer constitutional limits on shareholder voting, the absence of poison pills, and weaker golden parachutes, boards of directors can more easily remove CEOs (Bebchuk et al. (2009)). Indeed, Cornelli et al. (2013) find that a legal change which enhanced board authority over CEO replacement more than quadrupled the likelihood of forced CEO turnover.³²

We next discuss four specific implications on variation in board aggressiveness and CEO turnover. We note that the features discussed above regarding board compensation and composition may be suitable proxies for board aggressiveness. We first make a prediction concerning the heterogeneity of talent in the CEO labor market. Recall that, in Section 3, $\Delta\theta = \theta_H - \theta_L$ represents the dispersion in CEO ability, and can be identified with the

³²Cornelli et al. (2013) also show that more aggressive replacement ultimately resulted in improvements in firm value.

variance in CEO talent, $\text{Var}(\theta) = \pi(1 - \pi)(\Delta\theta)^2$, when π is fixed. Similarly, $\Delta = \bar{\theta} - \underline{\theta}$ in Section 4 also measures the variance $\text{Var}(\theta) = \Delta^2/12$. Our results in both sections imply that shareholders prefer a more aggressive board when the ability of the replacement is less dispersed.

Prediction 2. *Boards are more aggressive and there is greater CEO turnover in industries with a lower dispersion of talent in the CEO labor market distribution.*

There are several characteristics that contribute to dispersion in CEO talent. First, the variance is heightened in industries with CEO labor markets which entail greater uncertainty over candidates. For example, firms in industries which require significant firm-specific information face greater uncertainty in their CEO appointments—hiring a top executive from a similar firm may not lead to similar performance due to firm-specific policies or knowledge. The results thus predict that these industries will be met with relatively less CEO turnover. Likewise, as noted by Gao et al. (2017), firms which have deeper internal candidate pools may face less uncertainty over the new CEO appointment. They find that a deeper internal pool for the CEO position is associated with higher rates of CEO turnover. Lastly, the variance in talent naturally depends on the degree of homogeneity among firms in an industry. As Parrino (1997) argues, there is less variation in the replacement CEO when the new CEO is hired from a peer firm in a relatively homogeneous industry. He has found some evidence consistent with the above implication: CEO turnover is higher in industries which are more homogeneous.

Second, we consider how the noise in the performance measure, gauged by $\text{Var}(\varepsilon)$, affects board aggressiveness and CEO turnover frequency. The results imply that the board is *more* aggressive in replacing the CEO when the CEO's ability is more difficult to assess given the performance of the firm.

Prediction 3. *Boards are more aggressive and CEO turnover is greater in firms or industries where CEO ability is more difficult to assess.*

Existing research related to this prediction has found evidence which is consistent. For example, in a sample of leveraged buyouts, Cornelli and Karakaş (2015) find that CEO turnover decreases in the second phase of acquisition and is lower than the corresponding rate for public firms. This is consistent with the above prediction as private equity firms are very closely involved in the firm's operations, resulting in less uncertainty in their inferences of the CEO's ability. Additionally, a number of studies have found that private firms replace their CEO less often than public firms (e.g., Lel et al. (2014), Gao et al. (2017)). Private firms generally have more concentrated, closely held, and illiquid ownership, which results in

stronger monitoring incentives and hence less uncertainty over assessments of CEO ability (Kahn and Winton (1998)).

Third, shareholders prefer a more aggressive board when the pool of low types, $1 - \pi$ in Section 3, is high. We can interpret π as the prevalence of talented managers within an industry's labor market. For example, high growth or riskier product market industries, where CEOs are generally given more high-powered incentives, may tend to attract more talented managers. In contrast, in more stable or declining industries, we expect average CEO talent of the labor market distribution to be lower. Additionally, the concentration of manager types can be related to industry performance; in poorly performing industries we expect the board to be more aggressive in replacing the CEO.

Prediction 4. *Boards are more aggressive and there is greater CEO turnover in industries with lower average talent of the CEO labor market distribution.*

Some evidence for this prediction has been found by Kaplan and Minton (2012) and Jenter and Kanaan (2015). In particular, Jenter and Kanaan (2015) find that poor peer performance increases the likelihood that the CEO is dismissed. Our results help to explain this finding as poor peer performance (i.e., low π) is accompanied by more aggressive replacement decisions, leading to greater CEO turnover.

As the last implication on the variation in board aggressiveness, we predict that the board becomes more aggressive as the gap narrows between the personal benefit of remaining in power, χ , and the gain from effective communication, d . As discussed previously, this change positively affects the optimal aggressiveness of the board. However, its effect on CEO turnover is mixed. When this gap is narrow enough, the CEO is naturally more inclined to tell the truth, and thus shareholders can easily discourage less talented CEOs from overstatements and miscommunication through a more aggressive board. With a lower incidence of mimicry, insofar as the report $\hat{\theta}$ is sufficiently promising, the board may become *more* lenient in retaining CEOs with mediocre performance y . The aggressive board, however, is *less* lenient to CEOs who could not envision a strong future in the report: CEOs will be replaced unless $\hat{\theta}$ is high enough. These two opposite attitudes make the direction of the overall effect on CEO turnover frequency ambiguous. However, when lower types are heavily concentrated, the latter effect will dominate and increase CEO turnover frequency. The above argument leads to the following implications on board aggressiveness and CEO turnover.

Prediction 5. *Boards are more aggressive in firms or industries where CEO communication with the board is more valuable. CEO turnover is also greater in such firms or industries when there is a large concentration of lower ability CEOs in the CEO labor market.*

Prediction 6. *Boards are more aggressive in firms or industries which have lower CEO benefits. CEO turnover is also greater in such firms or industries when there is a large concentration of lower ability CEOs in the CEO labor market.*

Prediction 5 concerns the salience of the board's advisory capacity. In terms of more concretely classifying firms and industries where the board's advisory role is prominent, Klein (1998) and Coles et al. (2008, 2012) suggest that this is true for firms which are more complex, such as firms which are more diversified, larger, or more highly leveraged. For example, multi-segmented firms which have operations in different industries or segments may rely more on the board's advising capacity, as the board generally includes directors who are experts in different industries (Hermalin and Weisbach (1988), Yermack (1996)). Moreover, Markarian and Parbonetti (2007), Coles et al. (2008, 2012), and Linck et al. (2008) find evidence consistent with the notion that complexity of the industry or firm is also met with a greater advisory role of the board. In addition, the board's advisory role is more salient for firms which have a higher incidence of acquisitions or other major corporate actions (Paul (2007)). Hence, we predict that boards are set to be more aggressive when informative communication with the board is more valuable (high d), such as for firms with greater internal or external complexity, or for firms that engage more frequently in major corporate actions, such as acquisitions or divestitures.

This prediction is consistent with a number of empirical findings. Taylor (2010) finds that large firms—a proxy used for complexity by Coles et al. (2008)—exhibit excessive replacement of the CEO, consistent with the prediction above. Moreover, Weisbach (1988), Dahya et al. (2002), Huson et al. (2004) and Guo and Masulis (2015) have found a positive association between outside director presence on the board and CEO turnover, and that complex firms appoint more outside directors (e.g., Coles et al. (2008, 2012)). With regard to Prediction 6 on CEO benefits, the empirical literature has developed various proxies for CEO private benefits (e.g., Fos and Jiang (2015)).

In addition, our equilibrium analysis shows that the board employs a two-step retention policy. As the first step, the board asks the CEO to report her current situation as a message $\hat{\theta}$. If the CEO reports something too pessimistic, the board helps her to revive the firm during her tenure but the removal of the CEO is unchangeable. After having passed the first step, the CEO needs to achieve an output that surpasses the target set by the board (i.e., $y \geq k^*$) as the second step of the retention policy. We find in this model that low type managers often misreport soft information to the board, capturing the anecdotal evidence discussed in the introduction that CEOs often misrepresent inside information to board members (see also fn. 2). The significance of soft information is also well-documented by Cornelli et al. (2013), who show that boards often utilize unverifiable information when

making replacement decisions. Cornelli et al. (2013) note: “Hard information is neither necessary nor sufficient for boards to conclude that a CEO is incompetent. In nearly half (46.4%) of the cases in which boards first express the opinion that the CEO is incompetent, they do so following a year in which the firm met expectations (28.2%) or even outperformed (18.2%)” (p. 452). Our results formally capture this paradoxical phenomenon that boards sometimes remove CEOs with seemingly strong performance. In our setting, this feature endogenously emerges through the board’s reliance on soft information.

Our results on the two-stage retention policy provide implications regarding variation in the economic magnitude of the inverse relationship between performance and CEO turnover found in the empirical literature (e.g., Coughlan and Schmidt (1985), Warner et al. (1988), Kaplan (1994), Faleye et al. (2011), Jenter and Lewellen (2017)). In particular, a more aggressive board is *less* reliant on the hard information of CEO performance y , because the soft information conveyed in the first-step message $\hat{\theta}$ is highly indicative of the CEO’s ability when the board is aggressive enough to discourage message inflation. We therefore predict a *weaker* relationship between observed performance and CEO turnover as the board becomes more aggressive.

Prediction 7. *There is a weaker relation between observed performance and CEO turnover in firms or industries where boards are more aggressive.*

6 Discussions

In this section, we discuss alternative formulations to highlight important assumptions of this model. In Section 6.1, we first examine a model without replacement to observe how the presence of a replacement decision differentiates our benchmark setting from other communication models typically without replacement decisions. We then discuss an alternative approach of contractual design in Section 6.2.

6.1 Continuously many choices on a non-retention issue

We amend our baseline model to make the board decision comparable to that of Adams and Ferreira (2007) and Harris and Raviv (2005, 2008, 2010). Specifically, in this alternative formulation, the model has only one period, and at the end of the period, the board chooses an action $z \in \mathbb{R}$, instead of whether to retain the CEO. The variable z can be interpreted as, for example, the scale of a project as in Harris and Raviv (2005). The CEO’s payoff is $y - (\theta + b - z)^2$, where $b > 0$ represents the CEO’s bias in the board’s ultimate decision. The board’s objective is to maximize $y - (\theta - c - z)^2$. Note that the aggressiveness parameter

c now measures the discrepancy between the objectives of the board and the CEO; like our primary model, a higher value of c implies greater misalignment between the two players in the board's decision.

No other feature of the baseline model is altered. In particular, the CEO observes her type θ and reports $\hat{\theta}$ to the board. The board, in return, gives non-strategic advice to the CEO and the CEO chooses an action a . Lastly, the board observes output y (defined as before) and then chooses the non-retention decision z . The game ends without a retention or replacement decision for the CEO.

We examine whether aggressiveness or friendliness facilitates truthful communication from the CEO to the board, focusing on the case of two types as in our baseline model. (As before, $c > 0$ corresponds to an aggressive board and $c < 0$ corresponds to friendliness.) To begin, we briefly calculate the optimal choice of the board, before investigating the CEO's incentive. The decision problem of the board is to minimize the expected value of the quadratic loss $(\theta - c - z)^2$ by controlling z . When the board believes that the type of the CEO is θ_H with probability μ , the optimal choice is $z(\mu; c) = \mu\theta_H + (1 - \mu)\theta_L - c$.

Rationally anticipating the board's response, the low-type CEO compares the truthful message $\hat{\theta} = \theta_L$ and the misreport $\hat{\theta} = \theta_H$. If the low-type CEO truthfully reveals her type θ_L , the board chooses $z(0; c) = \theta_L - c$ regardless of the realization of y . Thus, this CEO's payoff from truth-telling is

$$U = \theta_L - (b + c)^2.$$

If the CEO inflates the report by choosing $\hat{\theta} = \theta_H$, the board's decision is also inflated to $z(\mu(y; \sigma); c) = \theta_L + \mu(y; \sigma) \cdot \Delta\theta - c$, where $\mu(y; \sigma)$ is, as in the text, the posterior probability of the high type after observing y when the low type inflates the message with probability σ . As a result, the payoff of mimicry is

$$U^m = (\theta_L - d) - \int_{-\infty}^{\infty} [b + c - \mu(y; \sigma) \cdot \Delta\theta]^2 f(y - \theta_L + d) dy.$$

We impose $b > \Delta\theta$ and $b + c \geq \Delta\theta$ as regularity conditions so that the low-type CEO always benefits from improvement in the belief μ .

We now examine the role of aggressiveness in the communication between the CEO and the board. We focus on the case that the low-type CEO is indifferent between the two messages so that the relative benefit of mimicking the high type, defined as $V = U^m - U$, is zero. In this case, we can examine how board aggressiveness influences the equilibrium

value of σ by investigating properties of the net mimicry value V :

$$\frac{\partial \sigma^*}{\partial c} = -\frac{\partial V / \partial c}{\partial V / \partial \sigma}$$

by the implicit function theorem. The denominator

$$\frac{\partial V}{\partial \sigma} = -2 \cdot \int_{-\infty}^{\infty} \left(-\frac{\partial \mu}{\partial \sigma} \right) \cdot [b + c - \mu(y; \sigma) \cdot \Delta \theta] f(y - \theta_L + d) dy$$

is negative because $\partial \mu / \partial \sigma$ is negative. The numerator is positive:

$$\begin{aligned} \frac{\partial V}{\partial c} &= 2(b + c) - 2 \cdot \int_{-\infty}^{\infty} [b + c - \mu(y; \sigma) \cdot \Delta \theta] f(y - \theta_L + d) dy \\ &= 2\Delta \theta \cdot \int_{-\infty}^{\infty} \mu(y; \sigma) f(y - \theta_L + d) dy > 0. \end{aligned} \quad (9)$$

These calculations imply that $\partial \sigma^* / \partial c$ is positive; i.e., an aggressive board *discourages* truthful communication in this alternative specification. This result, the opposite of the result found in the baseline model, is indeed consistent with the literature on friendly boards (e.g., Adams and Ferreira, 2007; Harris and Raviv, 2005, 2008, 2010).

To better understand this result, we consider the analogous condition for the baseline model studied in the text. Let $k^*(\sigma, c)$ be the optimal cutoff choice by the board given the mimicry probability σ and the aggressiveness parameter c . The concrete value of $k^*(\sigma, c)$ is given by the right-hand side of equation (3), which is increasing in both σ and c . The CEO's payoff from misreporting is

$$\begin{aligned} U_{base}^m &= (\theta_L - d) + \int_{-\infty}^{\infty} \chi \cdot \mathbf{1}_{\{y \geq k^*(\sigma, c)\}} f(y - \theta_L + d) dy \\ &= (\theta_L - d) + \chi \cdot \left\{ 1 - F(k^*(\sigma, c) + d - \theta_L) \right\} \end{aligned}$$

(plus a constant), while the payoff of truth-telling is simply $U_{base} = \theta_L$.³³ Thus, the net mimicry value for the baseline model is

$$V_{base} = U_{base}^m - U_{base} = \left\{ 1 - F(k^*(\sigma, c) + d - \theta_L) \right\} \cdot \chi - d.$$

Unlike the above extension, the net mimicry value is decreasing in both σ and c . In particular, the sign of $\partial V_{base} / \partial c$, which is negative, differs from that of $\partial V / \partial c > 0$.

³³In addition to either payoff appearing in this sentence, the CEO receives χ at the beginning of the first period and θ in the second period.

The difference in the signs of these partial derivatives highlights how our baseline model works differently from board models which do not consider CEO replacement. In our baseline model, an increase in c negatively affects the mimicry payoff U_{base}^m but has no effect on the truth-telling payoff U_{base} . In other words, an aggressive board reduces the gain from misreporting without becoming more punitive to truthful messages from low-ability CEOs than a neutral board. Essentially, this occurs because aggressiveness emerges only through a harsher attitude toward CEO replacement in our baseline model: while misreporting low-type CEOs suffer from the board’s aggravated attitude, truthful low-type CEOs ($\hat{\theta} = \theta_L$) endure no additional disutility in this regard—a neutral board is already maximally hostile (i.e., $k = \infty$) to them in retention—with payoffs from receiving advice unaffected. In this sense, widened misalignment in preferences between the CEO and the board encourages truthful communication (the disciplining effect). In other words, *misalignment in preferences* eventually leads to *alignment in incentives*—both the CEO and the board concentrate on maximally benefiting from effective communication—through disabling the source of private benefit of survival with the aggressive replacement policy. This point is very different from this alternative model without CEO replacement, as well as from other strategic communication models which find that greater misalignment in preferences deteriorates communication (e.g., Chakraborty and Yilmaz (2017)).

In the alternative setup with non-retention decisions, aggressiveness negatively affects both the truth-telling payoff U and the mimicry payoff U^m . Indeed, truth-telling CEOs suffer even more than misreporting ones (i.e., $\partial U/\partial c = -2(b + c) < \partial V/\partial c$), as shown in equation (9). This occurs because, by inflating the report, the low-type CEO can successfully manipulate the decision variable z upward. The increase in z alleviates the negative effects from the difference in interests, $b + c$, for the positively biased CEO. In other words, in this alternative setup, the CEO can let the board choose a “less biased” choice (from the CEO’s positively biased point of view) at the cost of forgoing informative advice from the board. As a result, ironically, the CEO suffers from board aggressiveness more in the case of truth-telling and lower z (more negatively biased, for the CEO) than in the case where both the report and z are inflated. The above argument is summarized in Table 1.

The above discussion highlights the importance of the board’s disciplinary role to determine the continuation of the CEO. As summarized in Table 1, in the model with the disciplinary role, an aggressive board negatively affects the payoff of misreporting CEOs but has no effect on truth-telling CEOs. However, with non-replacement decisions, board aggressiveness affects both truth-telling and misreporting CEOs, and indeed, truth-telling CEOs are more severely affected than misreporting ones. The last feature—more severe punishments for truth-telling CEOs—is consistent with the finding of Adams and Ferreira

	replacement decisions (baseline model)	non-replacement decisions (alternative model)
Truth-telling		
Board's decision	unchanged from $k = \infty$	<i>greater bias</i>
CEO's payoff	not affected	<i>negatively affected</i>
Misreporting		
Board's decision	<i>more strict</i>	<i>greater bias, but less biased</i>
CEO's payoff	<i>negatively affected</i>	<i>less severely affected</i>

Table 1: Effects of aggressive boards on low-type CEOs in the baseline model with replacement decisions and in the alternative model with non-replacement decisions.

(2007) that the board intervenes more often with truthful information disclosure than without it, and this tendency is worsened as the board becomes less friendly (see Section B.3 in Adams and Ferreira (2007)).³⁴ From this observation, we conclude that the replacement decision and thus the disciplinary role of the board is an indispensable feature to obtain the aggressive board result in this paper's framework.

6.2 Contract design and its challenges

In line with the extant literature that studies CEO communication with a board of directors, we do not assume commitment by the board to punish or reward the CEO (Adams and Ferreira (2007), Harris and Raviv (2005, 2008, 2010), Baldenius et al. (2014), Chakraborty and Yilmaz (2017)). This allows us to more precisely depict the contrast of our results with the existing literature on board-CEO communication. Nevertheless, it is potentially possible to design intricate contracts in which the compensation to the CEO could depend on various details of firm operations, including messages sent to the board. Such contracts, however, may face practical challenges. As noted by Adams and Ferreira (2007), "due to the limited time they spend in the firm, directors may not know what information they need, which makes it difficult for them to implement such contracts" (p. 223).

A similar concern about the contract-design approach arises in the current setting as well. The board in our setting could induce truthful revelation by offering a contract with a bundle of contingent clauses that specify how the CEO's report $\hat{\theta}$ of her productivity θ could affect the stream of payments to the CEO. As discussed in Section 2, the one-dimensional report $\hat{\theta}$ is meant to be a reduced-form representation of multi-dimensional information about various

³⁴Also, in delegation models such as Harris and Raviv (2005, 2008, 2010), an aggressive board policy to deprive the firm operation from the CEO makes it difficult for the CEO to truthfully communicate with the board, due to the cheap-talk structure.

aspects of the firm's current operations, each of which is indicative of the ability and fitness of the CEO. As such, at the time the contract is offered, the CEO has not yet learned this information, and thus may not know her fitness within the firm at the outset. Moreover, these presumably intricate links between details and productivity of the CEO may not be well understood when the contract is offered, compared to the moment of reporting when the CEO has already started her involvement in the firm's operations. Therefore, in the specific context of this paper, the board is realistically unlikely to be capable of preparing a well-designed contract that connects details of the report with the compensation of the CEO at the time when the offer is made.

We note that an additional issue with a contracting approach here is that our baseline model does not include the central components for considering agency (such as moral hazard). Adding these components may limit the tractability of the analysis and shift the focus of the model towards issues related to agency rather than on communication. However, a commitment approach may engender interactions between the CEO's communication incentives and the agency problem, which are not analyzed in the current setting. In this sense, such an approach could be an interesting avenue for future research.

7 Conclusion

This study investigates the CEO replacement capacity of the board and its effect on the communication between management and the board. The CEO must communicate her private information to the board, but she may be inclined to misreport for a better chance of continuation. In the unique equilibrium we have found, the board demands high standards in *both* reports and outputs. The board sets a minimally acceptable standard for the report and the performance measure, whereby the board determines the retention of the CEO. In the value-maximizing solution, we find that the board often sets an inefficiently aggressive replacement policy that minimizes the propensity of mimicry by low-ability managers. Consequently, faced with a low likelihood of retention, managers with low ability forgo their chances at surviving in the firm and instead turn to maximizing performance during their tenure. This aggressive replacement practice follows from the paradoxical equilibrium property that shareholders prefer to amplify the misalignment between the board and the CEO in order to improve the CEO's incentives in reporting. We refer to this property as the *disciplinary effect* of an aggressive board. Under an aggressive board, unfit managers are highly likely to be dismissed, along with some talented ones with bad luck.

Our notion of an aggressive board corresponds to one with stronger misalignment in preferences and a replacement policy that removes certain CEOs more often than would be

desirable ex post. This equilibrium policy is in contrast with much of the existing literature which typically finds that friendly, or management-aligned, boards are optimal for improving communication between the CEO and the board. The key distinction of our setting is that we examine the interplay between the advising and disciplinary functions of the board and its effect on CEO-board communication. The key economic force that drives our results is that, through an aggressive board, shareholders are able to punish misreporting CEOs without hurting truth-telling ones. Moreover, aggressiveness is effective in removing weak CEOs even though managers above the bar are also often erroneously dismissed. An aggressive board improves the value of the firm in the short term through informative communication, while replacing weak CEOs to improve future value.

The baseline model can be extended in several directions. To focus on the relationship between advising and replacement, we have assumed that the CEO cannot make an effort decision. Including an effort decision by the manager may lead to additional interesting results. CEOs who will be fired for sure will have their effort incentive reduced; however, board aggression can induce greater effort by managers in the misreporting region, as they may exert more effort to counteract the loss in informative communication. This can improve their output measure and make retention more likely. Hence, it is unclear how imposing an effort decision affects the optimality of board aggressiveness. Another direction in which the model can be explored is through an endogenous entrenchment mechanism that the CEO can impose, such as a long-lived project that is tied to the CEO's presence in the firm. This may lead to more or less aggression depending on how costly it is for the CEO to entrench herself, as well as the additional opportunity loss of the project to shareholders from removal. Finally, in our baseline setting, we assume that informative advising benefits the firm under each CEO type equally. One potentially interesting extension is to allow the board to determine the efficacy of advising through a costly effort decision, thereby endogenizing the advising benefit from informative communication, as in Laux (2017), who also allows advising to be heterogeneously useful to different CEO types. This extension could lead to additional implications regarding the interaction between value-increasing communication and replacement.

References

- ADAMS, R. B. AND D. FERREIRA (2007): “A theory of friendly boards,” *The Journal of Finance*, 62, 217–250.
- ALMAZAN, A. AND J. SUAREZ (2003): “Entrenchment and severance pay in optimal governance structures,” *The Journal of Finance*, 58, 519–547.
- BALDENIUS, T., N. MELUMAD, AND X. MENG (2014): “Board composition and CEO power,” *Journal of Financial Economics*, 112, 53–68.
- BEBCHUK, L., A. COHEN, AND A. FERRELL (2009): “What matters in corporate governance?” *The Review of financial studies*, 22, 783–827.
- BOONE, A. L., L. C. FIELD, J. M. KARPOFF, AND C. G. RAHEJA (2007): “The determinants of corporate board size and composition: An empirical analysis,” *Journal of financial Economics*, 85, 66–101.
- CASAMATTA, C. AND A. GUEMBEL (2010): “Managerial legacies, entrenchment, and strategic inertia,” *The Journal of Finance*, 65, 2403–2436.
- CHAKRABORTY, A. AND B. YILMAZ (2017): “Authority, consensus, and governance,” *The Review of Financial Studies*, 30, 4267–4316.
- CHO, I.-K. AND D. M. KREPS (1987): “Signaling games and stable equilibria,” *The Quarterly Journal of Economics*, 102, 179–221.
- COLES, J., N. DANIEL, AND L. NAVEEN (2012): “Board advising,” *Working paper*.
- COLES, J. L., N. D. DANIEL, AND L. NAVEEN (2008): “Boards: Does one size fit all?” *Journal of financial economics*, 87, 329–356.
- CORNELLI, F. AND O. KARAKAŞ (2015): “CEO turnover in LBOs: The role of boards,” *Working paper*.
- CORNELLI, F., Z. KOMINEK, AND A. LJUNGQVIST (2013): “Monitoring managers: Does it matter?” *The Journal of Finance*, 68, 431–481.
- COUGHLAN, A. T. AND R. M. SCHMIDT (1985): “Executive compensation, management turnover, and firm performance: An empirical investigation,” *Journal of accounting and economics*, 7, 43–66.
- CRAWFORD, V. P. AND J. SOBEL (1982): “Strategic information transmission,” *Econometrica: Journal of the Econometric Society*, 1431–1451.
- CRÉMER, J. (1995): “Arm’s length relationships,” *The Quarterly Journal of Economics*, 110, 275–295.
- DAHYA, J., J. J. MCCONNELL, AND N. G. TRAVLOS (2002): “The Cadbury committee, corporate performance, and top management turnover,” *The Journal of Finance*, 57, 461–483.
- DEMB, A. AND F. NEUBAUER (1992): *The corporate board: Confronting the paradoxes*, Oxford University Press.
- DENIS, D. J. AND D. K. DENIS (1995): “Performance changes following top management dismissals,” *The Journal of finance*, 50, 1029–1057.
- DEWATRIPONT, M. AND J. TIROLE (1994): “A theory of debt and equity: Diversity of securities and manager-shareholder congruence,” *The quarterly journal of economics*, 109, 1027–1054.
- DIKOLLI, S. S., W. J. MAYEW, AND D. NANDA (2014): “CEO tenure and the performance-turnover relation,” *Review of accounting studies*, 19, 281–327.
- DOMINGUEZ-MARTINEZ, S., O. H. SWANK, AND B. VISSER (2008): “In defense of boards,” *Journal of Economics & Management Strategy*, 17, 667–682.
- DOW, J. (2013): “Boards, CEO entrenchment, and the cost of capital,” *Journal of Financial Economics*, 110, 680–695.
- DYCK, A. AND L. ZINGALES (2004): “Private benefits of control: An international comparison,” *The journal of finance*, 59, 537–600.
- ELLIS, J., L. GUO, AND S. MOBBS (2016): “Do directors learn from forced CEO turnover experience,” *Social Science Research Network*.
- FALEYE, O. (2007): “Classified boards, firm value, and managerial entrenchment,” *Journal of Financial Economics*, 83, 501–529.
- FALEYE, O., R. HOITASH, AND U. HOITASH (2011): “The costs of intense board monitoring,” *Journal of Financial Economics*, 101, 160–181.
- FICH, E. M. AND A. SHIVDASANI (2012): “Are busy boards effective monitors?” in *Corporate Governance*, Springer, 221–258.
- FOS, V. AND W. JIANG (2015): “Out-of-the-money CEOs: Private control premium and option exercises,”

- The Review of Financial Studies*, 29, 1549–1585.
- GAO, H., J. HARFORD, AND K. LI (2017): “CEO Turnover–Performance Sensitivity in Private Firms,” *Journal of Financial and Quantitative Analysis*, 52, 583–611.
- GREGOR, M. AND B. MICHAELI (2020): “How CEO-Friendly Should Boards With Limited Attention Be?” *Working paper*.
- GUO, L. AND R. W. MASULIS (2015): “Board structure and monitoring: New evidence from CEO turnovers,” *The Review of Financial Studies*, 28, 2770–2811.
- HARRIS, M. AND A. RAVIV (2005): “Allocation of decision-making authority,” *Review of Finance*, 9, 353–383.
- (2008): “A theory of board control and size,” *The Review of Financial Studies*, 21, 1797–1832.
- (2010): “Control of corporate decisions: shareholders vs. management,” *The Review of Financial Studies*, 23, 4115–4147.
- HENNES, K., A. LEONE, AND B. MILLER (2007): “Management intent and CEO and CFO turnover around earnings restatements: Evidence from the post-Enron era,” *Working paper*.
- HERMALIN, B. E. AND M. S. WEISBACH (1988): “The determinants of board composition,” *The RAND Journal of Economics*, 589–606.
- (1998): “Endogenously chosen boards of directors and their monitoring of the CEO,” *American Economic Review*, 96–118.
- (2012): “Information disclosure and corporate governance,” *The Journal of Finance*, 67, 195–233.
- HUANG, S. AND G. HILARY (2018): “Zombie board: Board tenure and firm performance,” *Journal of Accounting Research*, 56, 1285–1329.
- HUSON, M. R., P. H. MALATESTA, AND R. PARRINO (2004): “Managerial succession and firm performance,” *Journal of Financial Economics*, 74, 237–275.
- HUSON, M. R., R. PARRINO, AND L. T. STARKS (2001): “Internal monitoring mechanisms and CEO turnover: A long-term perspective,” *The Journal of Finance*, 56, 2265–2297.
- INDERST, R. AND H. M. MUELLER (2010): “CEO replacement under private information,” *The Review of Financial Studies*, 23, 2935–2969.
- JENSEN, M. C. (1993): “The modern industrial revolution, exit, and the failure of internal control systems,” *the Journal of Finance*, 48, 831–880.
- JENTER, D. AND F. KANAAN (2015): “CEO turnover and relative performance evaluation,” *The Journal of Finance*, 70, 2155–2184.
- JENTER, D. AND K. A. LEWELLEN (2017): “Performance-induced CEO turnover,” *Working paper*.
- KAHN, C. AND A. WINTON (1998): “Ownership structure, speculation, and shareholder intervention,” *The Journal of Finance*, 53, 99–129.
- KAPLAN, S. N. (1994): “Top Executives, Turnover, and Firm Performance in Germany,” *Journal of Law, Economics, & Organization*, 142–159.
- KAPLAN, S. N. AND B. A. MINTON (2012): “How has CEO turnover changed?” *International review of Finance*, 12, 57–87.
- KARTIK, N. (2009): “Strategic communication with lying costs,” *The Review of Economic Studies*, 76, 1359–1395.
- KARTIK, N., M. OTTAVIANI, AND F. SQUINTANI (2007): “Credulity, lies, and costly talk,” *Journal of Economic Theory*, 134, 93–116.
- KATZ, R. (2003): “Former CEO misled board,” *Bloomberg News*, February 25, 2003.
- KLEIN, A. (1998): “Firm performance and board committee structure,” *The Journal of Law and Economics*, 41, 275–304.
- LARCKER, D. AND B. TAYAN (2016): “We Studied 38 Incidents of CEO Bad Behavior and Measured Their Consequences,” *Harvard Business Review*, June 9, 2016.
- LAUX, V. (2008): “Board independence and CEO turnover,” *Journal of Accounting Research*, 46, 137–171.
- (2014): “Corporate governance, board oversight, and CEO turnover,” *Foundations and Trends® in Accounting*, 8, 1–73.
- (2017): “Supporting and assessing agents,” *Journal of Accounting Research*, 55, 995–1016.
- LEL, U., D. MILLER, AND N. REISEL (2014): “Differences in agency problems between public and private firms: Evidence from top management turnover,” *Working paper*.
- LINCK, J. S., J. M. NETTER, AND T. YANG (2008): “The determinants of board structure,” *Journal of*

- financial economics*, 87, 308–328.
- LORSCH, J. W. AND E. MACIVER (1989): *Pawns or Potentates: The Reality of America's Corporate Boards*, Harvard Business School Press Boston, MA.
- MACE, M. L. G. ET AL. (1971): *Directors: Myth and reality*, Harvard Business School Press Boston, MA.
- MARKARIAN, G. AND A. PARBONETTI (2007): "Firm complexity and board of director composition," *Corporate governance: an international review*, 15, 1224–1243.
- MOBBS, S. (2013): "CEOs under fire: The effects of competition from inside directors on forced CEO turnover and CEO compensation," *Journal of Financial and Quantitative Analysis*, 48, 669–698.
- PARRINO, R. (1997): "CEO turnover and outside succession: A cross-sectional analysis," *Journal of Financial Economics*, 46, 165–197.
- PAUL, D. L. (2007): "Board composition and corrective action: evidence from corporate responses to bad acquisition bids," *Journal of Financial and Quantitative Analysis*, 42, 759–783.
- SONG, F. AND A. V. THAKOR (2006): "Information control, career concerns, and corporate governance," *The Journal of Finance*, 61, 1845–1896.
- TAYLOR, L. A. (2010): "Why are CEOs rarely fired? Evidence from structural estimation," *The Journal of Finance*, 65, 2051–2087.
- The Economist* (2001): "The fading appeal of the board room," March 31, 2001.
- WARNER, J. B., R. L. WATTS, AND K. H. WRUCK (1988): "Stock prices and top management changes," *Journal of financial Economics*, 20, 461–492.
- WARTHER, V. A. (1998): "Board effectiveness and board dissent: A model of the board's relationship to management and shareholders," *Journal of Corporate Finance*, 4, 53–70.
- WEISBACH, M. S. (1988): "Outside directors and CEO turnover," *Journal of financial Economics*, 20, 431–460.
- YERMACK, D. (1996): "Higher market valuation of companies with a small board of directors," *Journal of financial economics*, 40, 185–211.

Appendix

A Technical issues on equilibrium and signaling

A.1 Strategies and equilibrium

We impose a standard innocuous measure-theoretic restriction on strategies. After observing $\hat{\theta}^1$, the board must use a Borel-measurable *retention policy* $z : \mathbb{R} \rightarrow [0, 1]$. The value $z(y)$ represents the probability that the board retains the manager after observing output y (in addition to the message $\hat{\theta}^1$). This condition ensures that the manager can properly evaluate the expected value of each message for her (i.e., if the board used a non-measurable policy, the CEO cannot calculate expected values).³⁵ Also note that we allow the board to randomize retention and replacement for each value of y (in a measurable manner).

In defining the perfect Bayesian equilibrium, we impose sequential rationality for *all* realizations of history. In contrast, due to the measure-theoretic problem associated with conditioning, we require the posterior belief $\beta(\cdot|\hat{\theta})$ after message $\hat{\theta}$ to have the properties that define conditional probability only on a set $\hat{\Theta} \subseteq \Theta$ of messages that are reached with probability 1. We can specify the posterior belief after observing output y more naturally because y has a density $f(y - \theta + d \cdot \mathbf{1}_{\{\theta \neq \hat{\theta}\}})$. Specifically, after seeing y , the board must assign probability

$$\beta(S|\hat{\theta}, y) = \frac{\int_S f(y - \theta + d \cdot \mathbf{1}_{\{\theta \neq \hat{\theta}\}}) d\beta(\theta|\hat{\theta})}{\int_{\Theta} f(y - \theta + d \cdot \mathbf{1}_{\{\theta \neq \hat{\theta}\}}) d\beta(\theta|\hat{\theta})}$$

on a Borel-measurable set $S \subseteq \Theta$. We do not have to specify beliefs for period 2 because the board never makes a retention decision in that period.

A.2 Signaling and equilibrium selection criteria

We interpret the present paper's model as a signaling game as follows. The CEO is the sender and the board is the receiver. The type and message spaces are identical and given by a set Θ , which is either $\{\theta_L, \theta_H\}$ or $(\underline{\theta}, \bar{\theta})$. The action space for the receiver is the set of all the Borel-measurable retention policies $z : \mathbb{R} \rightarrow [0, 1]$. Recall that the action space includes all of the behavioral strategies for the board (after observing some message).

When Θ contains continuously many types, it is an obstacle in defining the D1 criterion that the definition of out-of-equilibrium messages is not self-evident because most (if not all) of the messages are chosen with probability 0. Nevertheless, we often encounter messages that we can naturally endorse as on-equilibrium messages. In particular, if some single type chooses some message with positive probability, then this message should be on equilibrium even if the chance that this message is chosen is 0 in the entire game. Motivated by the above argument, we define a *clearly on-equilibrium message* as a message that is chosen by

³⁵As we will see later, on the equilibrium path, optimal retention policies will be in the form of a *cutoff rule*; i.e., there is some *cutoff* $k \in [-\infty, +\infty]$ such that $z(y) = 1$ for $y > k$ and $z(y) = 0$ for $y < k$.

some type with positive probability. We define an *out-of-equilibrium message* as a message that is not clearly on equilibrium.

After defining out-of-equilibrium messages, we can define the D1 and D2 criteria (Cho and Kreps (1987)). Let $Z^*(\hat{\theta})$ be the set of retention policies that are a best response for the board with some non-degenerate belief (i.e., a probability distribution where no single point has probability 1) over type θ after observing message $\hat{\theta}$.³⁶ A perfect Bayesian equilibrium *survives the D1 criterion* if for all out-of-equilibrium messages $\hat{\theta}$, the posterior belief $\beta(\cdot|\hat{\theta})$ assigns no probability on types $\theta \in \Theta$ that satisfy the following condition: There exists a type $\theta^* \in \Theta$ such that $u_s(\theta, \hat{\theta}, z) \geq u_s^*(\theta)$ implies $u_s(\theta^*, \hat{\theta}, z) > u_s^*(\theta^*)$ for all $z \in Z^*(\hat{\theta})$. Here, $u_s(\theta, \hat{\theta}, z)$ is the payoff that the CEO with type θ and message $\hat{\theta}$ receives when the board uses retention policy z . Also, $u_s^*(\theta)$ is the equilibrium payoff for type θ . A perfect Bayesian equilibrium *survives the D2 criterion* if for all out-of-equilibrium messages $\hat{\theta}$, the posterior belief $\beta(\cdot|\hat{\theta})$ assigns no probability on types $\theta \in \Theta$ that satisfy the following condition: For all $z \in Z^*(\hat{\theta})$, there exists a type $\theta_z^* \in \Theta$ such that $u_s(\theta, \hat{\theta}, z) \geq u_s^*(\theta)$ implies $u_s(\theta_z^*, \hat{\theta}, z) > u_s^*(\theta_z^*)$.

B Proofs

B.1 Proof of Theorem 1

Consider part (ii). Recall that the board receives the expected payoff of $\mathbb{E}[\theta] + c$ after replacing a CEO, whereas the payoff is at most θ_H and at least θ_L if the CEO is retained. Thus, the board always replaces the CEO when $\mathbb{E}[\theta] + c > \theta_H$. Similarly, for part (iii), the board always retains the CEO when $\mathbb{E}[\theta] + c < \theta_L$.

Hereafter, we focus on part (i). We first provide the value of \hat{c} , which is uniquely given by $\mu^*(\hat{c}) = \mu(k_0; 1)$. Since $\mu^*(c) = \{\mathbb{E}[\theta] + c - \theta_L\} / \Delta\theta$, we obtain $\mathbb{E}[\theta] + \hat{c} = \Delta\theta \cdot \mu(k_0; 1) + \theta_L \in (\theta_L, \theta_H)$.

We now show the uniqueness of equilibrium, assuming that (A1) the high type θ_H always reports the truthful message; and (A2) after the low message θ_L , the board believes that the manager is a low type for sure. We later show that these two assumptions must hold in any D1 equilibrium (Lemmas 4 and 5). First suppose $\mu^*(c) \leq \mu(k_0, 1)$. If the mimicking probability σ^* is less than 1, the equilibrium cutoff k^* must be k_0 or greater by (6) and consequently, $\mu(k^*; \sigma^*) > \mu(k_0, 1) \geq \mu^*(c)$. That is, this value of k is suboptimal and thus σ^* must be 1. With $\sigma^* = 1$, the optimal cutoff for the board is given by $\mu(k; 1) = \mu^*(c)$ and the equilibrium cutoff k^* must be identical to the unique solution of this equality condition.

³⁶The restriction to non-degenerate beliefs is needed only to obtain Lemma 8, which eliminates the possibility that the board assigns probability one on type $\theta_1 = \mathbb{E}[\theta] + c$ and employs some pathological retention policy. This possibility significantly complicates our application of the D1 and D2 criteria and lets the proof of Lemma 10 fail.

There are several alternative approaches that can eliminate the above problem. We can achieve the desired result by imposing a behavioral assumption that the board chooses either choice with probability 1 whenever indifferent. Alternatively, we can treat type θ_1 as simply nonexistent. It is also possible to restrict the board to cutoff rules. It also works to discretize the type space such that type θ_1 is genuinely nonexistent. In any way, non-cutoff retention policies (after report $\hat{\theta} > \theta_1$) cannot survive minor fluctuations of the model and thus we should naturally eliminate them as in Lemma 8.

Second, suppose $\mu^*(c) > \mu(k_0, 1)$. If $\sigma^* = 1$, then the equilibrium cutoff k^* is given by $\mu(k^*; 1) = \mu^*(c)$. Since $\mu^*(c) > \mu(k_0, 1)$, we have $k^* > k_0$. This means $\sigma^* = 0$ by (6). This is a contradiction. Therefore, $\sigma^* < 1$ and $k^* = k_0$ hold in equilibrium. To make k^* optimal for the board, σ^* must satisfy $\mu(k_0; \sigma^*) = \mu^*(c)$ and this condition uniquely determines the value of σ^* .

Now we apply the D1 criterion to eliminate implausible equilibria. In particular, we need to preclude an equilibrium where both types choose message θ_L . In such an equilibrium, the pecking order of the productivity may be reversed; i.e., $\theta_H - d$ can be less than θ_L . In this case, as shown in the next lemma, the board employs a *reversed cutoff rule*: the board replaces (retains) the manager if $y > k_L$ ($y < k_L$, respectively) with some cutoff k_L .

Lemma 3. *Consider the board's optimal retention policy for message θ_L . When $\theta_H - d > \theta_L$, the optimal policy is a cutoff rule with a unique optimal cutoff $k_L \in [-\infty, +\infty]$. When $\theta_H - d < \theta_L$, the optimal policy is a reversed cutoff rule with a unique optimal cutoff $k_L \in [-\infty, +\infty]$.*

Proof. If the board assigns probability p on θ_H after observing message θ_L , then the posterior probability on θ_H after observing output y in addition is

$$\mu_L(y) = \frac{pf(y - \theta_H - d)}{pf(y - (\theta_H - d)) + (1 - p)f(y - \theta_L)} = \frac{p}{p + (1 - p)Q(y)}$$

with $Q(y) = f(y - \theta_L)/f(y - (\theta_H - d))$.

First suppose $p \in (0, 1)$. When $\theta_H - d > \theta_L$, the likelihood ratio Q is increasing and the posterior belief μ_L is decreasing. Thus, a cutoff rule is optimal. When $\theta_H - d < \theta_L$, the likelihood ratio Q is decreasing and the posterior belief μ_L is increasing. In this case, the optimal strategy needs to be a reversed cutoff rule. In either case, the optimal cutoff k_L is uniquely determined by $\mu_L(k_L)\theta_H + (1 - \mu_L(k_L))\theta_L = \mathbb{E}[\theta] + c$, or equivalently, $\mu_L(k_L) = \mu^*(c)$.

When $p \in \{0, 1\}$, the board either always replaces the manager (when $p = 0$) or always retains her (when $p = 1$). We can represent these policies as reversed and regular cutoff strategies with an extreme cutoff $k_L \in \{-\infty, +\infty\}$. \square

The next lemma proves that the the D1 criterion implies the assumption (A1). In particular, there is no D1 equilibrium where both types pool on the bad message θ_L .

Lemma 4. *The manager with type θ_H truthfully reports her type in any equilibrium that survives the D1 criterion.*

Proof. We first eliminate the case that type θ_H sends message θ_L with probability $p \in (0, 1)$. We show that type θ_L strictly prefers message θ_L to θ_H . If this is the case, the board retains the manager with report θ_H regardless of the output because type θ_H is the only type that chooses message θ_H . With this retention policy, type θ_H strictly prefers message θ_H ; a contradiction. (In this case, we do not need the D1 criterion because both messages are used with positive probability.)

First suppose $\theta_H - d > \theta_L$. In this case, the board uses a cutoff rule with cutoff k_L for message θ_L . The type θ_H is indifferent between the two messages only if the retention probability $F(\theta_H - d - k_L)$ for θ_L equals the retention probability $F(\theta_H - k_H)$ for θ_H ; i.e., $k_L = k_H - d$. Since k_L is lower, type θ_L clearly prefers the truthful report.

Second suppose $\theta_H - d < \theta_L$. The indifference condition for type θ_H is $k_L + d - \theta_H = \theta_H - k_H$ due to the reversal. Here, k_L is the cutoff for the equilibrium reversed cutoff rule for message θ_L . The retention probability for type θ_L is $F(k_L - \theta_L)$, which is greater than $F(\theta_L - d - k_H)$ because $k_L - \theta_L = 2\theta_H - \theta_L - d - k_H > \theta_L - d - k_H$. Again, type θ_L prefers message θ_L .

Finally, when $\theta_H - d = \theta_L$, both types face the same retention probability with message θ_L . When the message is θ_H , type θ_L has a lower retention rate than type θ_H . Hence, type θ_L prefers the truthful message in this case as well.

We now turn to the case that both types choose θ_L for sure. To apply the D1 criterion, we show that, whenever the board employs a retention policy for message θ_H that is a best response with *some* belief, type θ_L weakly prefers the message θ_H only if type θ_H strictly prefers that message. If this is the case, the D1 criterion prunes the possibility that type θ_L sends message θ_H and the board assigns probability 1 on type θ_H after observing message θ_L . With this belief, the board always retains the manager after observing message θ_H and thus the manager with type θ_H truthfully reports her type. This is a contradiction.

We consider the three cases once again. First suppose $\theta_L < \theta_H - d$. Type θ_L weakly prefers message θ_H only if $k_H > k_L + d$ and type θ_H strictly prefers message θ_H . Second, when $\theta_L > \theta_H - d$, type θ_L weakly prefers message θ_H only if $\theta_L - k_H > k_L + d - \theta_L$. The retention probability for type θ_H is $F(\theta_H - k_H)$ with message θ_H and $F(k_L + d - \theta_H)$ with message θ_L . The former is greater because $\theta_H - k_H > k_L + d + \theta_H - 2\theta_L > k_L + d - \theta_H$. Thus, type θ_H prefers the truthful report. Finally, when $\theta_L = \theta_H - d$, both types face the same retention probability with message θ_L . Hence, whenever type θ_L weakly prefers message θ_H , type θ_H strictly prefers that message. \square

We then show that the assumption (A2) holds even when no type chooses message θ_L .

Lemma 5. *Suppose that the manager sends message θ_H regardless of her type in an equilibrium that survives the D1 criterion. Then, the board assigns probability 1 on type θ_L after observing message θ_L .*

Proof. To apply the D1 criterion, we show that, whenever the board employs a retention policy for message θ_L that is a best response with *some* belief, type θ_H weakly prefers the message θ_L only if type θ_L strictly prefers that message. If this is the case, the D1 criterion prunes the possibility that type θ_H sends message θ_L and the board assigns probability 1 on type θ_L after observing message θ_L .

We consider the three cases once again. First, suppose $\theta_H - d > \theta_L$. Suppose that the board uses cutoff $k_L \in [-\infty, +\infty]$ for message θ_L . Type θ_H weakly prefers message θ_L only if the retention probability is higher with message θ_L than with the truthful message; i.e., $k_L + d > k_H$ in this case. If this is the case, type θ_L strictly prefers the truthful message.

Second, consider the case of $\theta_H - d < \theta_L$ and let k_L be the cutoff (of a reversed cutoff rule) for message θ_L . Type θ_H weakly prefers message θ_L only if $F(k_L - \theta_H + d) > F(\theta_H - k_H)$, or equivalently, $k_L - \theta_H + d > \theta_H - k_H$. The retention probability for type θ_L is $F(k_L - \theta_L)$ with message θ_L and $F(\theta_L - d - k_H)$ with message θ_H . The former is greater because $k_L - \theta_L > 2\theta_H - \theta_L - d - k_H > \theta_L - d - k_H$.

Finally, when $\theta_H - d = \theta_L$, the two types face the same retention probability with message θ_L . Since type θ_H has a higher equilibrium retention rate than type θ_L , type θ_L strictly prefers message θ_L whenever type θ_H weakly prefers message θ_L . \square

Finally, we show the existence of a D1 equilibrium. Let σ^* and k^* as in the statement of this theorem. We show that the following strategies constitute an equilibrium: the manager with type θ uses report θ_H with probability 1 when $\theta = \theta_H$ and with probability σ^* when $\theta = \theta_L$; the board uses cutoff k^* after observing report θ_H and cutoff $+\infty$ after report θ_L ; and the board believes that the manager is a low type after observing message θ_L . We have already shown that this equilibrium survives the D1 criterion by Lemma 5: the message θ_L is the only message that can be out of equilibrium, and if it is the case, the type that survives for this message is θ_L (the D1 criterion does not eliminate all the types). The optimality of (σ^*, k^*) follows from the best response conditions. After message θ_L , the board optimally chooses cutoff $+\infty$ because of the most pessimistic belief. The high type optimally reports the true type for this extreme cutoff following the low message θ_L .

B.2 Proof of Lemma 1

On the interval $[\hat{c}, \theta_H - \mathbb{E}[\theta]]$, the equilibrium cutoff k is a constant k_0 and the probability σ is decreasing in c . Since the normalized objective function $V(\sigma, k)$ is decreasing in σ with $k = k_0$ fixed, the objective function is increasing in c . The maximum value on this domain is achieved at the maximum value $c = \theta_H - \mathbb{E}[\theta]$.

B.3 Proof of Lemma 2

On the interval $(\theta_L - \mathbb{E}[\theta], \hat{c}]$, the equilibrium cutoff, implicitly given by $\mu(k; 1) = \mu^*(c)$, is increasing in c whereas σ is always equal to 1. When $\sigma = 1$, the objective function for the shareholders is $V(\sigma, k) = \pi(1-\pi)\Delta\theta \cdot W(k) - (1-\pi)d$, where $W(k) = F(\theta_H - k) - F(\theta_L - k - d)$. Note that

$$W'(k) = f(\theta_L - k - d) - f(\theta_H - k) = f(\theta_L - k - d) \left\{ 1 - \frac{f(\theta_H - k)}{f(\theta_L - k - d)} \right\}$$

changes the sign, from positive to negative, only once because of the monotone likelihood ratio property. In other words, the functions $W(k)$ and $V(1, k)$ have a single peak. Also, the first-order derivative hits 0 when the cutoff k is given as the equilibrium value with $c = 0$ as explained in the text. Therefore, $c = 0$ is optimal if it is included in the domain; if not, $c = \hat{c}$ is optimal.

B.4 Proof of Theorem 2

We first show that $\hat{c} \leq 0$ is equivalent to $m \leq m_*$. The condition $\hat{c} \leq 0$ is equivalent to $\mu(k_0; 1) \leq \mu^*(\hat{c})$, which in turn is equivalent to $R(k_0) \geq 1$ since $\mu^*(\hat{c}) = \pi$. Note that $R(k_0) \equiv f(m)/f(m - 2m_*) = 1$ occurs only when $m = m_*$. Since the ratio $f(m)/f(m - 2m_*)$ is decreasing in m , the condition $R(k_0) \geq 1$ is equivalent to $m \leq m_*$.

When $m \leq m_*$, or equivalently $\hat{c} \leq 0$, the objective function V is increasing in c on $(\theta_L - \mathbb{E}[\theta], \theta_H - \mathbb{E}[\theta])$ and thus the optimal value of c is $\theta_H - \mathbb{E}[\theta]$. This result is part (i) of this theorem.

To show part (ii), suppose $m > m_*$, or equivalently $\hat{c} > 0$. As shown in the text, the value of the normalized objective function V at the first peak $c = 0$ is V_N and that at the

second-peak $c = \theta_H - \mathbb{E}[\theta]$ is V_A . Their difference

$$V_A - V_N = \Delta \cdot \pi(1 - \pi) \left\{ F(2m_* - m) - \left[2F(m_*) - 1 - \frac{d}{\pi \cdot \Delta\theta} \right] \right\},$$

is non-negative if and only if the condition in part (ii) of this theorem holds.

B.5 Proof of Corollary 1

As d approaches χ , or as χ approaches d , the value of $m = F^{-1}(1 - d/\chi)$ converges to $F^{-1}(0) = -\infty$, while $m_* = (\Delta\theta + d)/2$ stays positive. Hence, Condition (i) of Theorem 2 is satisfied.

As π or $\Delta\theta$ shrinks to zero, the fraction in inequality (7) goes to positive infinity. Thus, when $m > m_*$, Condition (ii) of Theorem 2 holds. If $m \leq m_*$, Condition (i) is satisfied. In either case, the optimal board is aggressive.

Lastly, we consider the case of σ_ε^2 . Let $\Phi(z)$ be the distribution function of the standard normal distribution. Since ε is normally distributed with variance σ_ε^2 , we have $F(x) = \Phi(x/\sigma_\varepsilon)$ and $F^{-1}(p) = \sigma_\varepsilon \Phi^{-1}(p)$. Consequently, we can express inequality (7) as

$$\Phi \left(\frac{2m_*}{\sigma_\varepsilon} - \Phi^{-1} \left(1 - \frac{d}{\chi} \right) \right) + \frac{d}{\pi \cdot \Delta\theta} \geq 2\Phi \left(\frac{m_*}{\sigma_\varepsilon} \right) - 1. \quad (\text{B.1})$$

As σ_ε goes to infinity, the left-hand side of (B.1) converges to a positive value, $\Phi(-\Phi^{-1}(1 - d/\chi)) + d/(\pi \cdot \Delta\theta)$, whereas the right-hand side tends to zero because $\Phi(m_*/\sigma_\varepsilon) \rightarrow \Phi(0) = 1/2$. Therefore, either Condition (i) or (ii) of Theorem 2 must be satisfied.

Internet Appendix

(For online publication)

C The model with continuous types

In this appendix, we study a model with continuously many types $(\underline{\theta}, \bar{\theta})$. Recall that the distribution of these continuous types has a density function $g(\theta)$. Before proceeding to the analysis, we provide a brief overview of the results in this appendix. With continuous types, the CEO's misreporting behavior is characterized by two thresholds, θ_1 and θ_2 , introduced below. Depending on the leniency parameter c , the board determines a threshold level θ_1 such that it prefers to remove any CEO whose type is below this cutoff. Consequently, types lower than the threshold θ_1 must inflate their report in order to have a chance to remain as CEO. Indeed, an aggressive board sets θ_1 higher than the average ability $\mathbb{E}[\theta]$, meaning that even types above the average have an incentive to misreport if they are below the threshold. However, not all of the lower types manipulate their reports. We find that the CEO reports truthfully when her productivity type is too low to effectively mimic a type above θ_1 . In particular, there is an endogenously determined threshold θ_2 such that CEO types below this threshold report truthfully. (The truthful reports from these low types entail certain replacement, which is unchanged from the model with two types.) This threshold θ_2 , as well as θ_1 , depend on the shareholders' board policy c . We provide conditions under which the shareholders prefer the board to be aggressive and then examine comparative statics. This analysis provides additional insights, such as predictions regarding the variation in CEO turnover under heightened performance uncertainty (variance of ε) and with greater uncertainty over the CEO's type (variance of θ).

C.1 Equilibrium with exogenous aggressiveness

As in the two-type setting, we begin the analysis of the continuous model where c is exogenously given and then examine the case of endogenously determined c . In the second period, the manager has no incentive to misreport. This occurs for the same reason as in the two-type model; there is no replacement decision at that point and hence the manager cannot benefit from misreporting. The action selected in period 2 by a CEO of type θ is thus $a^2 = \omega^2(\theta)$. In the ensuing analysis, we consider incentives in the first period.

Also, we focus on the case of $c \in [\underline{\theta} - \mathbb{E}[\theta], \bar{\theta} - \mathbb{E}[\theta]]$. As we have seen in the two-type model (Theorem 1), only trivial equilibria emerge with extremely large or small c .³⁷ We state the result for the alternative case and then switch to the primary case of $c \in [\underline{\theta} - \mathbb{E}[\theta], \bar{\theta} - \mathbb{E}[\theta]]$.

Proposition 2. *Suppose $c < \underline{\theta} - \mathbb{E}[\theta]$ or $c > \bar{\theta} - \mathbb{E}[\theta]$. Then, the CEO always reports her true type. The board always retains the CEO if $c < \underline{\theta} - \mathbb{E}[\theta]$. When $c > \bar{\theta} - \mathbb{E}[\theta]$, the board always replaces the CEO.*

³⁷The two exemplary equilibria stated in Proposition 2 survive both D1 and D2 criteria because there is no out-of-equilibrium message.

Structure of equilibria

Like the two-type model, the CEO does not necessarily report her true type in the first period. Suppose that the board believes that reports are truthful. Then, by replacing the manager with report θ , the board forgoes the period-2 expected output θ under the current manager but instead gains the subsidy or cost c and the expected period-2 output $\mathbb{E}[\theta]$ after replacement. Therefore, the board replaces the CEO when the report θ is less than $\mathbb{E}[\theta] + c$, whereas the CEO with $\theta > \mathbb{E}[\theta] + c$ remains in her position.

We naturally conjecture that the threshold $\theta_1 = \mathbb{E}[\theta] + c$ determines the behavior of the manager in the following manner. The manager with type $\theta > \theta_1$ truthfully reports her type to the board, presumably because she does not need to hide her type in order to survive in the current firm. On the other hand, the CEO with type $\theta < \theta_1$ sometimes reports a message higher than θ_1 in order to have a chance to remain in the firm.

Given that the reports above θ_1 pool different types, the board sometimes must replace the CEO such that productive managers are retained with a higher likelihood than less productive ones. As in the model with two types, the optimal retention policy is a cutoff rule.

The derivation for the optimal cutoff is somewhat involved, so we refer readers to Appendix C.4 for the technical details and discussion regarding equilibrium selection. We find that the board employs a uniform cutoff $k_* \in [-\infty, +\infty]$ for all reports it receives from the CEO above θ_1 . In order to show this, we first prove that all reports above θ_1 occur on the equilibrium path and that a CEO with type $\theta > \theta_1$ always reports truthfully. We state the result here and provide the details in Appendix C.4.

Proposition 3. *In any equilibrium that survives the D1 criterion, the manager with type above θ_1 truthfully reports her type, and the board (almost surely) employs an identical threshold level k_* for replacement after observing any report above θ_1 .*

Proposition 3 significantly simplifies the analysis in several ways. First, as mentioned above, all messages above θ_1 are on the equilibrium path. Hence, we no longer need to worry about equilibrium selection on these high messages. Second, we can partition the type space $\Theta = (\underline{\theta}, \bar{\theta})$ into two disparate intervals: types above θ_1 and types below. What remains is to analyze the behavior of CEO types in the interval $(\underline{\theta}, \theta_1]$.

Third, the manager with type $\theta \in (\underline{\theta}, \theta_1]$ will be replaced for sure unless she pretends to have a high type $\hat{\theta} \in (\theta_1, \bar{\theta})$. More precisely, it cannot occur (except on an event of probability 0) that the CEO has type $\theta \in (\underline{\theta}, \theta_1]$, reports $\hat{\theta} \in (\underline{\theta}, \theta_1]$, and is retained. Thus, we can essentially assume that types $\theta \in (\underline{\theta}, \theta_1]$ have only two choices: the truthful report θ or some misreport $\hat{\theta} \in (\theta_1, \bar{\theta})$. However, it is guaranteed only *on the equilibrium path* that the board replaces the manager for sure after message $\hat{\theta} \in (\underline{\theta}, \theta_1]$: the message $\hat{\theta} \in (\underline{\theta}, \theta_1]$ can be out of equilibrium, and after this message, the board may still form a modestly optimistic belief and set a cutoff that is not infinitely strict (i.e., $k(\hat{\theta}) < \infty$) but high enough to discourage every type from using this message. We ultimately show that such optimistic beliefs cannot survive equilibrium selection (Theorem 5). Meanwhile, we simply assume in the exposition that *any report $\hat{\theta} \in (\underline{\theta}, \theta_1]$ results in the removal of the manager for sure* (i.e., $k(\hat{\theta}) = +\infty$).

In sum, we have characterized the structure of equilibria for types and reports higher than θ_1 . It is still unclear if, as expected, the board removes the CEO after receiving a message below θ_1 . We postpone the analysis of this question as it requires another stage of equilibrium selection. We instead investigate the reporting behavior of manager types below θ_1 , assuming that any report below θ_1 certainly induces CEO replacement. We then return to the equilibrium selection problem (Theorem 5).

Equilibrium decisions

As in the model with two types, the manager with $\theta < \theta_1$ faces two choices. If the manager reports her true type, she obtains the informational gain d but the board removes her before the next period with probability one. By reporting something above θ_1 , the manager forgoes the gain d but has a positive chance to remain in the current firm. Since the probability of retention is $1 - F(k_* + d - \theta)$ in the latter case, the indifference condition is

$$\{1 - F(k_* + d - \theta)\} \chi = d.$$

By solving this equation, we obtain the threshold type with which the manager is indifferent between the above two choices:

$$\theta_2(k_*) = k_* + d + F^{-1}(d/\chi). \tag{C.1}$$

Types below this threshold θ_2 cannot gain a satisfactory retention rate even after misreporting (i.e., $\{1 - F(k_* + d - \theta)\} \chi < d$). These types thus rather prefer to report truthfully. In contrast, types above θ_2 but below θ_1 prefer to inflate their report because $\{1 - F(k_* + d - \theta)\} \chi > d$. The following lemma summarizes this argument.³⁸

Lemma 6. *Suppose $\theta < \theta_1$. In any equilibrium that survives the D1 criterion, the manager truthfully reports her type if $\theta < \theta_2$; and reports a message above θ_1 if $\theta > \theta_2$.*

The value of θ_2 is thus the threshold such that types below this level report truthfully and are replaced with certainty. As we see shortly, shareholders can induce informative communication (i.e., truthful reports) from types lower than θ_2 by raising this threshold θ_2 . This is achieved by setting a more aggressive board and consequently raising θ_1 (at the expense of misreporting by intermediate types). This feature is analogous to the *disciplinary effect* we observed in the two-type model.

We then investigate how the uniform cutoff k_* is determined by the board given this reporting behavior. We saw above that the cutoff levels $k(\hat{\theta})$ for reports $\hat{\theta} > \theta_1$ must be some uniform level k_* (Proposition 3), but each cutoff level $k(\hat{\theta})$ needs to be a solution of the optimization problem for the board and thus depends on the posterior belief after observing report $\hat{\theta}$. Thus, if the posterior beliefs for such reports are not properly aligned—e.g., when certain messages attract too many (or too few) misreporting types—the board may employ several different cutoffs, which never occurs in equilibrium due to Proposition 3. In what

³⁸We allow $\theta_2 > \theta_1$ and $\theta_2 < \underline{\theta}$. The latter case does not cause any problem as long as we set $g(\theta) = 0$ for $\theta < \underline{\theta}$. We can easily see that $\theta_2 > \theta_1$ never occurs in equilibrium; if k_* is so high that θ_2 exceeds θ_1 , the manager never misreports her type and thus k_* goes down to $-\infty$.

follows, we instead require the board to choose some uniform cutoff k_* for (almost) every $\hat{\theta} > \theta_1$ and find a necessary condition for each value of $\hat{\theta}$. In this way, we can eventually obtain a single, useful condition that determines the value of k_* as a function of θ_2 , and the function $k_*(\theta_2)$ works as if it is the board's best response function.

For ease of exposition, we focus on an equilibrium where all types in the misreporting interval (θ_2, θ_1) employ the same density function $h(\hat{\theta})$ in choosing a misreport $\hat{\theta}$.³⁹ We first calculate the posterior belief of the board after observing $\hat{\theta} \in (\theta_1, \bar{\theta})$:

$$\text{Prob} \left\{ \theta = \hat{\theta} \mid \hat{\theta} \right\} = \frac{g(\hat{\theta})}{g(\hat{\theta}) + h(\hat{\theta}) \int_{\theta_2}^{\theta_1} g(\theta) d\theta} = \frac{g(\hat{\theta})}{Q(\hat{\theta})}$$

and,

$$\text{Prob} \left\{ \theta \neq \hat{\theta} \mid \hat{\theta} \right\} = \frac{h(\hat{\theta}) \int_{\theta_2}^{\theta_1} g(\theta) d\theta}{g(\hat{\theta}) + h(\hat{\theta}) \int_{\theta_2}^{\theta_1} g(\theta) d\theta} = \frac{h(\hat{\theta}) \int_{\theta_2}^{\theta_1} g(\theta) d\theta}{Q(\hat{\theta})},$$

where $Q(\hat{\theta}) = g(\hat{\theta}) + h(\hat{\theta}) \int_{\theta_2}^{\theta_1} g(\theta) d\theta$ represents the probability (density) that the manager chooses report $\hat{\theta}$. Also note $\int_{\theta_2}^{\theta_1} g(\theta) d\theta$ is the unconditional probability of misreporting. More specifically, the posterior probability of $\theta \leq x$ is

$$\text{Prob} \left\{ \theta \leq x \mid \hat{\theta} \right\} = \frac{h(\hat{\theta})}{Q(\hat{\theta})} \int_{\theta_2}^x g(\theta) d\theta,$$

for all $x \in (\theta_2, \theta_1)$. Hence, type $\theta \in (\theta_2, \theta_1)$ has a density $h(\hat{\theta})g(\theta)/Q(\hat{\theta})$ conditional on report $\hat{\theta}$, while the truthful type $\theta = \hat{\theta}$ has probability $g(\hat{\theta})/Q(\hat{\theta})$ as an atom.

Given the above posterior belief, the board must be indifferent between keeping and replacing the manager after observing output $y = k_*$. The corresponding indifference condition is

$$\underbrace{\hat{\theta} \cdot \frac{f(k_* - \hat{\theta})g(\hat{\theta})}{Q(\hat{\theta})}}_{\text{conditional prob. of } \hat{\theta}} + \underbrace{\int_{\theta_2}^{\theta_1} \theta \cdot \frac{f(k_* + d - \theta)h(\hat{\theta})g(\theta)}{Q(\hat{\theta})} d\theta}_{\text{conditional density of } \theta} = \theta_1,$$

or equivalently,

$$(\hat{\theta} - \theta_1)f(k_* - \hat{\theta})g(\hat{\theta}) = h(\hat{\theta}) \int_{\theta_2}^{\theta_1} (\theta_1 - \theta)f(k_* + d - \theta)g(\theta) d\theta. \quad (\text{C.2})$$

This condition guarantees the optimality of cutoff k for each *individual* report $\hat{\theta}$.

By integrating this individual-level condition (C.2) with respect to $\hat{\theta}$, we obtain an *ag-*

³⁹Such an equilibrium always exists, but many other equilibria also exist. See the proof of Lemma 7 for the general case.

gregate necessary condition for optimality:

$$\int_{\theta_1}^{\bar{\theta}} (\theta - \theta_1) f(k_* - \theta) g(\theta) d\theta = \int_{\theta_2}^{\theta_1} (\theta_1 - \theta) f(k_* + d - \theta) g(\theta) d\theta. \quad (\text{C.3})$$

The left-hand side represents the aggregate positive effect from keeping the current manager across all types above θ_1 . The right-hand side is the corresponding effect from the misreporting types. When θ_2 is given, the equilibrium value of the uniform cutoff k_* must satisfy the necessary condition (C.3). Indeed, we uniquely find the value of k_* that solves (C.3) due to the monotone likelihood property. The following lemma summarizes the above argument and provides additional results.

Lemma 7. *In any equilibrium that survives the D1 criterion, the uniform cutoff k_* for reports above θ_1 satisfies condition (C.3). For each value of $\theta_2 \in [-\infty, \theta_1)$, there uniquely exists $k_*(\theta_2) \in \mathbb{R}$ that solves condition (C.3). The function $k_*(\theta_2)$ is continuous, non-increasing, and goes to $-\infty$ as $\theta_2 \rightarrow \theta_1$.*

To construct an equilibrium, we aim to find a pair (θ_2^f, k^f) that simultaneously satisfies $\theta_2^f = \theta_2(k^f)$ and $k^f = k_*(\theta_2^f)$. To this end, we consider a function

$$\Gamma(k) = \begin{cases} k_*(\theta_2(k)) & \text{if } \theta_2(k) < \theta_1 \\ -\infty & \text{otherwise,} \end{cases}$$

and its fixed point. The function Γ is non-increasing because k_* is non-increasing and θ_2 is increasing. The maximum $\Gamma(-\infty) = k_*(\underline{\theta})$ is finite and the minimum $\Gamma(+\infty) = k_*(\theta_1)$ goes to $-\infty$. Since Γ is continuous, we can find a unique fixed point k^f . The fixed point k^f is finite and thus θ_2^f is smaller than θ_1 .

Proposition 4. *The function $\Gamma(k)$ has a unique, finite fixed point k^f . Define $\theta_2^f = \theta_2(k^f)$. Then, $\theta_2^f < \theta_1$ and the pair (θ_2^f, k^f) satisfies $k^f = k_*(\theta_2^f)$ as well as $\theta_2^f = \theta_2(k^f)$.*

Due to Proposition 4, any equilibrium that survives the D1 criterion must use k^f as the uniform cutoff and θ_2^f as the threshold θ_2 of misreporting. Conversely, we can also construct an equilibrium from these two parameters. (Equation (C.2) constructs an equilibrium by determining the equilibrium value of $h(\hat{\theta})$.) We have fully characterized (except on the set of probability 0) the behavior of the board and the manager on the equilibrium path, but it remains unknown whether the board sets $k(\hat{\theta}) = +\infty$ after out-of-equilibrium messages between θ_2^f and θ_1 . Although this out-of-equilibrium behavior is now irrelevant in characterizing what happens on the equilibrium path, we can obtain the desired result—an infinite cutoff for bad messages—by imposing the D2 criterion.⁴⁰

Theorem 5. *Consider the model with exogenous $c \in [\underline{\theta} - \mathbb{E}(\theta), \bar{\theta} - \mathbb{E}(\theta)]$. There exists a perfect Bayesian equilibrium that survives both the D1 and D2 criteria. Any equilibrium that survives the D1 criterion almost surely satisfies the following properties in the first period:*

⁴⁰The D2 criterion imposes more restrictions than the D1 criterion. Thus, if an equilibrium survives the D2 criterion, then this equilibrium also survives the D1 criterion. See fn. 49 regarding why the D1 criterion is insufficient.

- (i) The manager truthfully reports her type if $\theta > \theta_1$ or $\theta < \theta_2^f$. The manager with type $\theta \in (\theta_2^f, \theta_1)$ chooses some report above θ_1 .
- (ii) The board replaces the manager when the manager reports $\hat{\theta} < \theta_1$ or the cash flow y is less than k^f . The board retains the manager if $\hat{\theta} > \theta_1$ and $k > k^f$.

Here, θ_2^f and k^f are as in Proposition 4. Furthermore, in any equilibrium that survives the D2 criterion, the board sets cutoff $+\infty$ after almost every report $\hat{\theta} \in (\underline{\theta}, \theta_1)$.

We make three remarks before proceeding to endogenize the parameter c . First, the CEO’s reporting behavior is non-monotonic as depicted in Figure 5. This non-monotonicity occurs because the CEO with low θ is unable to get a reasonably high chance of retention when she mimics some type. Hence, types below θ_2 truthfully reveal their ability, learn the true state, and then are subsequently replaced. The threshold type θ_2 ’s retention probability from mimicking is high enough that she is indifferent between truthful reporting and misreporting. The types above θ_2 but below θ_1 overstate their types, sometimes far above θ_1 . The types above θ_1 report truthfully.

Second, the board employs a two-step retention policy. As the first step, the manager asks the CEO to report her current situation. If the CEO reports something too pessimistic, the board helps her to revive the firm during her tenure but the removal of the CEO is unchangeable. This corresponds to the findings of Cornelli et al. (2013), who show that boards often utilize “soft” (nonverifiable) information regarding the CEO’s ability when making replacement decisions. Having passed the first step, to stay in the firm, the CEO needs to achieve the target k_* set by the board, as the second step. This equilibrium replacement behavior helps to explain the inverse relationship between performance and CEO turnover found in the empirical literature (see, e.g., Jenter and Lewellen (2017)).

Third, managers with intermediate ability $\theta \in (\mathbb{E}[\theta], \theta_1)$ are sometimes removed due to poor communication with the board (i.e., misreporting). Because truthful reporting terminates her tenure, these CEOs are urged to overstate their situation. This miscommunication reduces the effectiveness of the advice from the board and, consequently, CEOs with ability in this range tend to have worse performance due to the lack of information.

C.2 Shareholders’ decision

Due to the complexity of the model with a continuous type space, we must employ additional distributional assumptions in order to obtain analytic results with endogenously determined c . Specifically, we assume that type θ and noise ε are uniformly distributed on supports $[\underline{\theta}, \bar{\theta}]$ and $[-q, q]$, respectively. Although the uniform distribution $F(\varepsilon)$ does not fully satisfy the monotone likelihood ratio property, the distribution can be seen as a limit of distributions with this property.⁴¹ We note that the results are not qualitatively sensitive to these assumptions, as shown in the simulations reported in Appendix C.3.

One benefit of using the uniform distribution is that we can calculate closed-form characterizations of the board’s cutoff strategy and the shareholders’ optimal board policy. One

⁴¹Here is an example of such a sequence. Let ϕ denote the density of the standard normal distribution and define $z_n(x)$ by $z_n = |x|/n$ for $|x| < q$ and by $z_n = n|x| - q(n - 1/n)$ for other x . Then, density $f_n(x) = \phi(z_n(x))/\int_{-\infty}^{\infty} \phi(z_n(y)) dy$ satisfies the monotone likelihood ratio property, because z_n is convex, and the corresponding distribution $F_n(x) = \int_{-\infty}^x f_n(x) dx$ weakly converges to the uniform distribution.

drawback, however, of the uniform setting is that certain pathological cases arise which confound the analysis. We thus impose the following regularity conditions:

$$2q/\chi > \frac{4q - \Delta}{2\Delta}, \quad (\text{C.4})$$

where $\Delta = \bar{\theta} - \underline{\theta}$. This condition ensures that the distortion effect in the cutoff (i.e., too strict cutoff) is not too strong. We later see that $2q/\chi$ has a negative effect on $k - \theta_2$. When θ_2 is fully determined by other parameters—this is the case in Proposition 5 (i)—an increase in $2q/\chi$ lowers the level of equilibrium cutoff and thus mitigates the distortion in the retention decision. We also impose the following condition:

$$\underline{\theta} + q - (1 + 8q/\chi)^2 d \geq \bar{\theta} - q. \quad (\text{C.5})$$

This inequality is a technical condition that significantly simplifies the analysis by eliminating subtle pathological cases that arise in the dual uniform setting. To interpret this condition, note that the inequality (C.5) implies $\underline{\theta} + q > \bar{\theta} - q$; that is, the support of ε is sufficiently large such that low-type CEOs can potentially mimic up to the highest type.

We first characterize the equilibria with exogenously given c for this parameterization. We focus on equilibria consistent with the equilibria found in Appendix C.1: *The board employs a uniform cutoff k for messages above θ_1 and replaces the manager for sure with messages below θ_1 .* To simplify the notation, let $\theta_{2,0} = d + F^{-1}(d/\chi) \equiv d - q + 2dq/\chi$ denote the intercept of the threshold $\theta_2 = k + \theta_{2,0}$ (see equation (C.1)).

Proposition 5. *Consider the game described above with exogenously given c , and we focus on the class of equilibria described above. Assume the regularity conditions (C.4) and (C.5).*

- (i) *Suppose $c \in (0, \Delta/2)$. In any equilibrium, the board sets a uniform cutoff $k = 2\theta_1 - \bar{\theta} - \theta_{2,0}$ and the threshold $\theta_2 = 2\theta_1 - \bar{\theta}$ is greater than the worst type $\underline{\theta}$. The cutoff is high enough to replace even the best type with positive probability (i.e., $\bar{\theta} - q \leq k$).*
- (ii) *Suppose $c \in (-\Delta/2, 0)$. In any equilibrium, the board sets a uniform cutoff $k = 2\theta_1 - q - \underline{\theta}$ and no type below θ_1 chooses a truthful message (i.e., $\theta_2 \leq \underline{\theta}$). The cutoff is low enough such that the best type is never replaced (i.e., $\bar{\theta} - q \geq k$).*
- (iii) *Suppose $c = 0$. In any equilibrium, no type below θ_1 chooses a truthful message (i.e., $\theta_2 \leq \underline{\theta}$). The neutral board has continuously many optimal cutoffs and the set of optimal cutoffs is the interval between the cutoffs given in (i) and (ii); i.e., $[\bar{\theta} - q, \underline{\theta} - \theta_{2,0}]$.*
- (iv) *Suppose $|c| \geq \Delta/2$. In any equilibrium, the manager truthfully reports her type for sure. The board replaces the manager with probability 1 if $c \geq \Delta/2$. The manager is retained for sure if $c \leq -\Delta/2$.*

For any value of c , an equilibrium exists.

The first two cases are especially important. In case (i), types above θ_1 are rare so that the board needs to bring the cutoff k high enough to discourage lower types from mimicking. As a result, the worst types report truthfully but even the best type faces the risk of replacement. In equilibrium, the misreporting interval between θ_1 and θ_2 needs to be perfectly balanced with the types above θ_1 —due to the uniform specification—so that $\bar{\theta} - \theta_1 = \theta_1 - \theta_2$. This value of θ_2 , in turn, determines the value of $k = \theta_2 - \theta_{2,0}$.

In contrast, the board in case (ii) faces a large mass of types above θ_1 so that the cutoff is too low to discourage imitation. Consequently, some types above θ_1 are never replaced with the friendly choice of a cutoff. The threshold is type $\theta = k + q$; types above the threshold always have output higher than the cutoff. The equilibrium condition in this case is thus $(k + q) - \theta_1 = \theta_1 - \underline{\theta}$.

Cases (iii) and (iv) are less important in two different senses. Case (iv) is a trivial case where the board employs an extremely aggressive or friendly retention policy. The equilibrium multiplicity in case (iii) is apparently problematic, but the choice of k does not affect the payoff for the shareholders because the neutral board perfectly represents the shareholders' interest. Therefore, we only need to analyze the first two cases, keeping in mind that the extreme board (case (iv)) could be optimal. Indeed, we ultimately show that the optimal choice of c always lies in case (i).

We now aim to find the optimal level of c for shareholders. As in the two-type model, the shareholders' payoff can be divided into the two periods. In the first period, shareholders receive the payoff

$$V_1 = -d \{G(\theta_1) - G(\theta_2)\},$$

plus $\mathbb{E}[\theta]$. Here, the difference $G(\theta_1) - G(\theta_2) = \Pr\{\theta \in [\theta_2, \theta_1]\}$ is the probability of misreporting. The CEO in period 2 never misreports her type, but the board's retention policy in the first period affects the expected value of θ in the second period. The expected second-period value is

$$V_2 = \int_{\theta_1}^{\bar{\theta}} (\theta - \mathbb{E}[\theta]) F(\theta - k) g(\theta) d\theta + \int_{\theta_2}^{\theta_1} (\theta - \mathbb{E}[\theta]) F(\theta - k - d) g(\theta) d\theta, \quad (\text{C.6})$$

plus $\mathbb{E}[\theta]$. The shareholders maximize the sum $V = V_1 + V_2$ by controlling c .

We can analytically calculate the values of V_1 and V_2 in this dual uniform environment due to Proposition 5. In the exposition, we focus on the relevant case of $c \in (0, \Delta/2)$. (See the Appendix for the case of the friendly board.) Since $\theta_2 = 2\theta_1 - \bar{\theta} = \mathbb{E}[\theta] + 2c - \Delta/2$, the period-1 payoff V_1 is

$$V_1 = -d \left(\frac{1}{2} - \frac{c}{\Delta} \right). \quad (\text{C.7})$$

The period-2 payoff V_2 is similarly given as:

$$V_2 = \int_{\theta_1}^{\bar{\theta}} (\theta - \mathbb{E}[\theta]) \left(\frac{\theta - k + q}{2q} \right) \frac{d\theta}{\Delta} + \int_{\theta_2}^{\theta_1} (\theta - \mathbb{E}[\theta]) \left(\frac{\theta - d - k + q}{2q} \right) \frac{d\theta}{\Delta}. \quad (\text{C.8})$$

By $k = 2\theta_1 - \bar{\theta} - \theta_{2,0} = \mathbb{E}[\theta] + 2c - \Delta/2 - \theta_{2,0}$, we obtain the following cubic function:

$$V_2 = \frac{1}{48q} \left(1 - \frac{2c}{\Delta} \right) \{ 24c \cdot \theta_{2,0} + 4c(\Delta + 6q) + 2\Delta^2 - d(18c - 3\Delta) - 16c^2 \}. \quad (\text{C.9})$$

Note that $1 - 2c/\Delta > 0$ because $c < \Delta/2$.

Since $V = V_1 + V_2$ is a cubic function with a positive coefficient on c^3 , the function V will increase, attain a local maximum, decrease, and then increase again if V behaves regularly enough. Such non-monotonicity occurs due to the countervailing disciplinary and distortion effects. Recall that these two effects first emerged in the analysis of the two-type setting in Section 3. The disciplinary effect from raising c results in low-type CEOs reporting truthfully to the board. In this continuous setting, the distortion effect not only appears in the form of an excessively strict *cutoff*, but also emerges as an inefficiently high standard in *reporting*. That is, the board demands highly optimistic reports for retention considerations, and consequently, intermediate types $\theta \in (\mathbb{E}[\theta], \theta_1)$ must leave their position after truthful reporting.

After a few calculations, we obtain that the local maximum is attained at

$$c_A^* = \frac{1}{8} \left\{ d \left(1 + \frac{8q}{\chi} \right) + 2\Delta - \sqrt{D} \right\}, \quad (\text{C.10})$$

where $D = d^2(1 + 8q/\chi)^2 + 4d\Delta - 32dq + 4\Delta^2 > 0$. In the Appendix, we show that this local optimum is indeed the global optimum.

Theorem 6. *Assume the regularity conditions (C.4) and (C.5). It is optimal for shareholders to choose an aggressive board with $c \in (0, \Delta/2)$. The optimal value of c is uniquely given by equation (C.10).*

Theorem 6 states that shareholders set the board to be aggressive, and provides a closed-form characterization of the optimal policy c . We note that, in contrast to the two-type case, the optimal policy is not maximal aggression (i.e., $c \in (0, \Delta/2)$). We find that a moderately aggressive board is optimal since the shareholders' disutility from the distorted retention decision eventually exceeds the benefit of the disciplinary effect as the degree of aggressiveness, c , increases.

This result is in stark contrast to several theoretical studies which have found that a management-friendly board (excessive retention) is optimal for shareholders. By considering the interrelationship between advising and replacement, we find that aggressive boards (excessive replacement) can be optimal for shareholders, which upends the results of models that separately examine advising (e.g., Adams and Ferreira (2007)) or replacement (e.g., Almazan and Suarez (2003)).

Furthermore, the explicit solution c_A^* allows for comparative statics analysis. Note that the variables q and Δ are interchangeable with the variances of noise ε and type θ , respectively, in the comparative statics below, because $\text{Var}(\varepsilon) = q^2/3$ and $\text{Var}(\theta) = \Delta^2/12$.

Proposition 6. *Assume the regularity conditions (C.4) and (C.5). The optimal aggressiveness c_A^* is increasing in the cost of miscommunication, d , and in the variance of noise ε , and decreasing in the variance of type θ . An increase in the private benefits, χ , decreases (increases) c_A^* if $\Delta - d(8q - \Delta)$ is positive (negative).*

An increase in d , the value loss from uninformative communication, results in shareholders setting a more aggressive board. Intuitively, this occurs since shareholders prefer to elicit greater truthful reporting, and thus informative communication, from the CEO. Indeed, the

coefficient on c in V_1 increases (see equation (C.7)). On the other hand, an increase in d also *decreases* the cutoff k , as seen in Proposition 5 (i), by making truthful communication more attractive and misreporting less likely. Consequently, to compensate for this milder replacement threshold by the board and due to their strengthened preference for truthful reporting, shareholders push to make the board *more* aggressive in their replacement of the CEO as d increases.

Similarly, shareholders prefer a more aggressive board as the variance of noise (and thus q) increase. In this case, the support of ε expands and more low-type managers are potentially able to mimic a higher type in their observed output. Likewise, there is less room for very high types to meet the cutoff with certainty. As a result, the cutoff-based retention policy becomes less effective and the choice of cutoff k becomes less important (indeed, V_2 shrinks as q increases; see (C.9)). In other words, the increase in the noise dilutes the distortion effect of aggressiveness, while the disciplinary effect, represented by V_1 , is unchanged. With the disciplinary effect intact but the distortion effect weakened, shareholders face an incentive to make the board *more* aggressive in response to an increase in q .

Proposition 5 also shows that the shareholder's optimal aggressiveness c_A^* is decreasing in the variance of the CEO's productivity θ (and thus Δ). As the variance increases, the shareholders face an increased risk in replacing a highly talented CEO. We find that the increase in variance amplifies the disutility from the distortion effect of a high cutoff k , and leads shareholders to prefer a comparatively less aggressive board.

Lastly, the effect of an increase in the private benefits χ is negative when Δ (or equivalently, the variance of productivity θ) is sufficiently large compared to d and q (or equivalently, the variance of noise ε). As the private benefit χ increases, misreporting becomes more appealing for a low-type manager. The board, in turn, responds to the CEO's increased incentive for mimicry by increasing the retention standard k . The shareholders do not favor this decision of raising the bar when Δ is large enough. As already seen in the previous paragraph, an increase in Δ worsens the distortion effect of an aggressive board. Hence, the board's response to increasing the cutoff k is an *overreaction* from the shareholders' perspective, thus resulting in a *decrease* in c_A^* . Conversely, the distortion effect becomes relatively unimportant in a highly noisy situation (when q is high relative to Δ), which is also already seen two paragraphs above. This induces the shareholders to make the board more aggressive in response to the increased misreporting incentive from a higher χ .

C.3 Numerical Results

In this appendix, we present numerical exercises for additional economic implications and to show that the results of the model are robust to alternative distributions of θ and ε . We first assume that the noise ε is normally distributed and type θ is exponentially distributed. More specifically, the distribution of ε has mean 0 and variance 1 and the exponential distribution has intensity $\lambda = 1$. Also, we use $d = 1$ and $\chi = 2$ throughout the numerical analyses.

As shown in Figure 6, the optimal board is moderately aggressive (note that $\theta_1 = 1 + c$); the optimal value of c is numerically given as $c \approx 0.4775$. The bottom of the negative spike in Figure 6 represents the point where θ_2 reaches $\underline{\theta}$ ($= 0$). This sharp drop in the shareholders' payoff occurs due to frequent misreporting triggered by the board's excessively friendly retention policy. Figure 7 elucidates this point by describing how misreporting

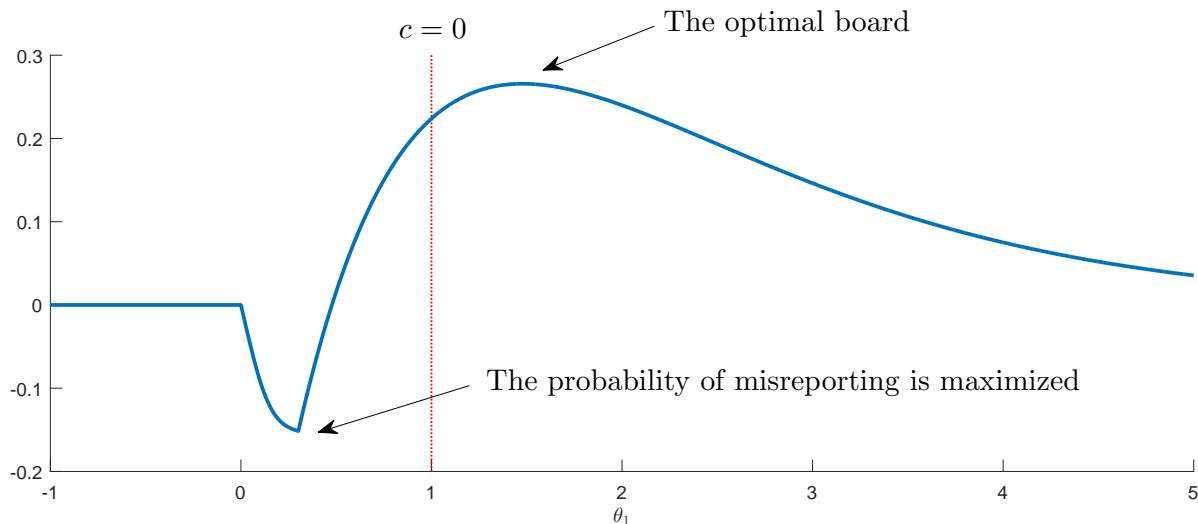


Figure 6: The objective function of the board.

behavior changes in response to a change in $\theta_1 = \mathbb{E}[\theta] + c$. The bottom of the negative spike in Figure 6 corresponds to the peak of the spike in the top panel of Figure 7 (i.e., when the misreporting probability is greatest). The misreporting region and the interval initially increase as θ_1 increases, and is maximized when θ_2 reaches $\underline{\theta}$ ($= 0$). After this point, the misreporting interval $[\theta_2, \theta_1]$ is pushed leftward and exponentially reduces its probability while keeping the width $\theta_2 - \theta_1$ constant.⁴² This disciplinary effect—the reduction of misreports due to the board’s aggressive retention policy—creates the hump after the negative spike in Figure 6 and induces a moderately aggressive board to be optimal for shareholders.

To see how the other effect—distortion in retention decisions—hurts the value of the firm for shareholders, see Figure 8 which depicts the contribution of the equilibrium retention policy to the shareholders’ objective function. To gauge the level of distortion in the equilibrium retention policy, we introduce the *optimal cutoff* $k^{\text{opt}}(\theta_1, \theta_2)$ with the misreporting interval $[\theta_2, \theta_1]$ as given. Formally, the optimal cutoff $k^{\text{opt}}(\theta_1, \theta_2)$ is the solution of the maximization problem

$$\int_{\theta_1}^{\bar{\theta}} (\theta - \mathbb{E}[\theta])F(\theta - k)g(\theta) d\theta + \int_{\theta_2}^{\theta_1} (\theta - \mathbb{E}[\theta])F(\theta - k - d)g(\theta) d\theta.$$

The corresponding first-order condition

$$\int_{\theta_1}^{\bar{\theta}} (\theta - \mathbb{E}[\theta])f(\theta - k^{\text{opt}})g(\theta) d\theta = \int_{\theta_2}^{\theta_1} (\mathbb{E}[\theta] - \theta)f(\theta - k^{\text{opt}} - d)g(\theta) d\theta,$$

⁴²Interestingly, Figure 7 shows that, while the probability of misreporting declines as θ_1 increases, the misreporting interval $[\theta_2, \theta_1]$ remains constant after the maximum is reached. This occurs because the board is shifting the misreporting interval $[\theta_2, \theta_1]$ further away from the high-density regions populated with low types.

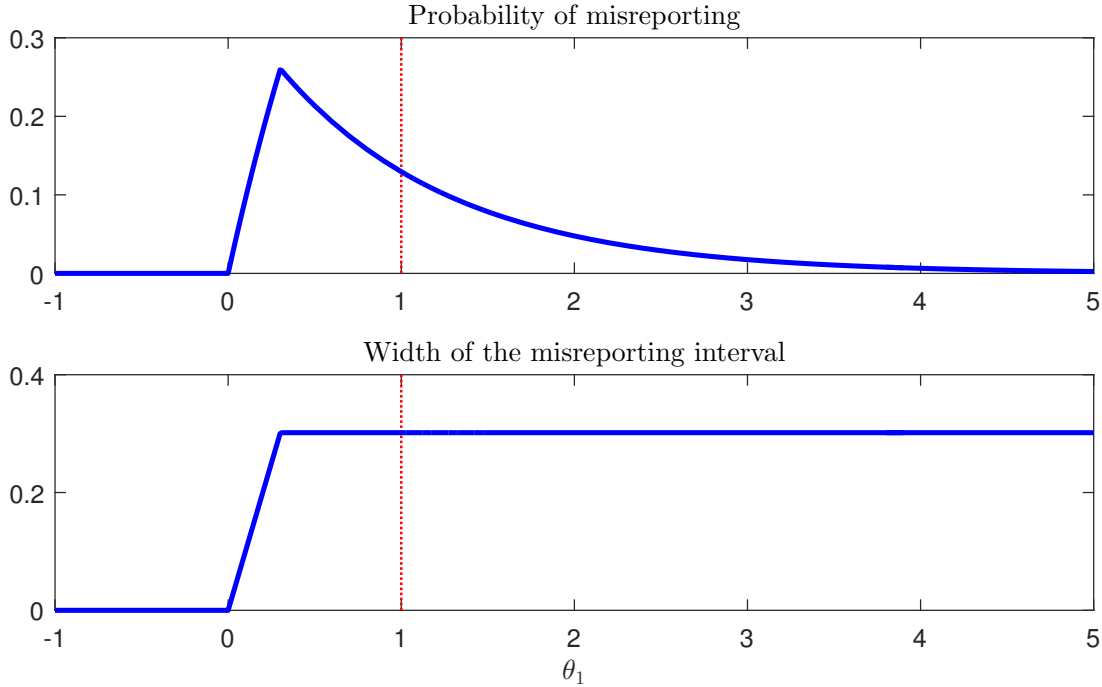


Figure 7: Two variables that capture the misreporting behavior of the manager: the probability of misreporting (i.e., $G(\theta_1) - G(\theta_2)$) and the width of the misreporting interval $[\max\{\theta_2, \underline{\theta}\}, \theta_1]$.

resembles but differs from the equilibrium condition (C.3), in which the board uses $\theta_1 = \mathbb{E}[\theta] + c$ in place of $\mathbb{E}[\theta]$. As shown in the lower half of Figure 8, the equilibrium cutoff is suboptimal unless $c = 0$: the board is too strict in setting k when aggressive (i.e., $c > 0$) and too lenient when friendly (i.e., $c < 0$).

Although Figure 8 shows that the equilibrium cutoff significantly deviates from the optimal level, this figure also suggests that the effect to the value of the firm is limited on the aggressive side (i.e., $c > 0$). To see why this is the case, we examine how the board's aggressive cutoff impacts the second-period value V_2 of the firm, defined in equation (C.6). We decompose this effect by considering three groups: the truth-telling top group $(\theta_1, \bar{\theta})$, misreporting intermediate group $(\mathbb{E}[\theta], \theta_1)$, and misreporting bottom group $(\theta_2, \mathbb{E}[\theta])$. (The types below θ_2 are the real bottom group, but they are replaced for sure anyway and thus not affected by the uniform cutoff.)

According to Figure 9, the suboptimality of the equilibrium cutoff slightly affects the bottom group $(\theta_2, \mathbb{E}[\theta])$ but has virtually no effect on the top and middle groups when $\theta_1 > \mathbb{E}[\theta]$ is close enough to $\mathbb{E}[\theta]$. Indeed, the effect on the bottom group is *positive* because a high cutoff helps to remove unwanted types in this group. Instead, the suboptimally high cutoff decreases the period-2 values from the top two groups, but the top group appears almost unaffected. The effect to the middle group is also minute (although it appears to be large due to the scaling of the graph). This observation does not change even if we replace the optimal cutoff with the first-best, but infeasible, retention policy: $k = +\infty$ for the bottom group and $k = -\infty$ for the top two groups. This observation implies that the

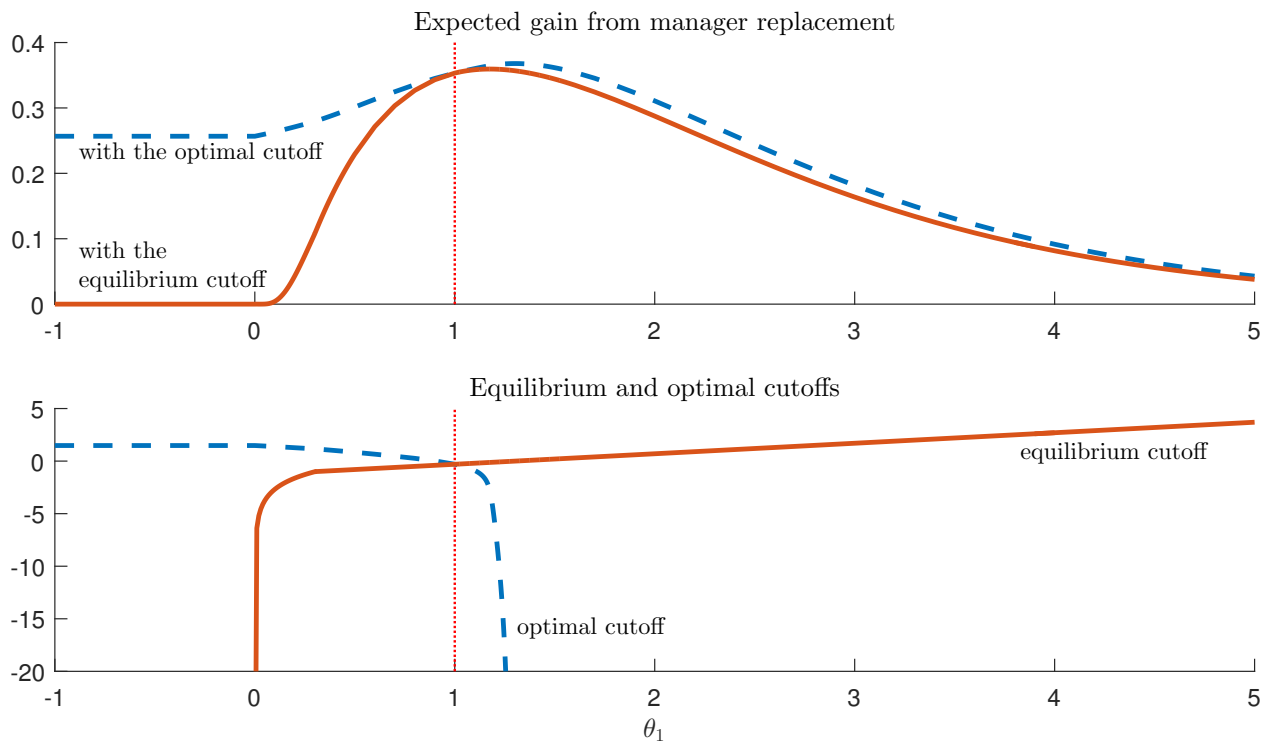


Figure 8: The gain from the manager selection ($V_2 - \mathbb{E}[\theta]$) and the choice of cutoff in the equilibrium retention policy. The solid curves represent the actual values in equilibrium. The dashed curves correspond to the optimal cutoff $k^{\text{opt}}(\theta_1, \theta_2)$ with θ_1 and θ_2 fixed.

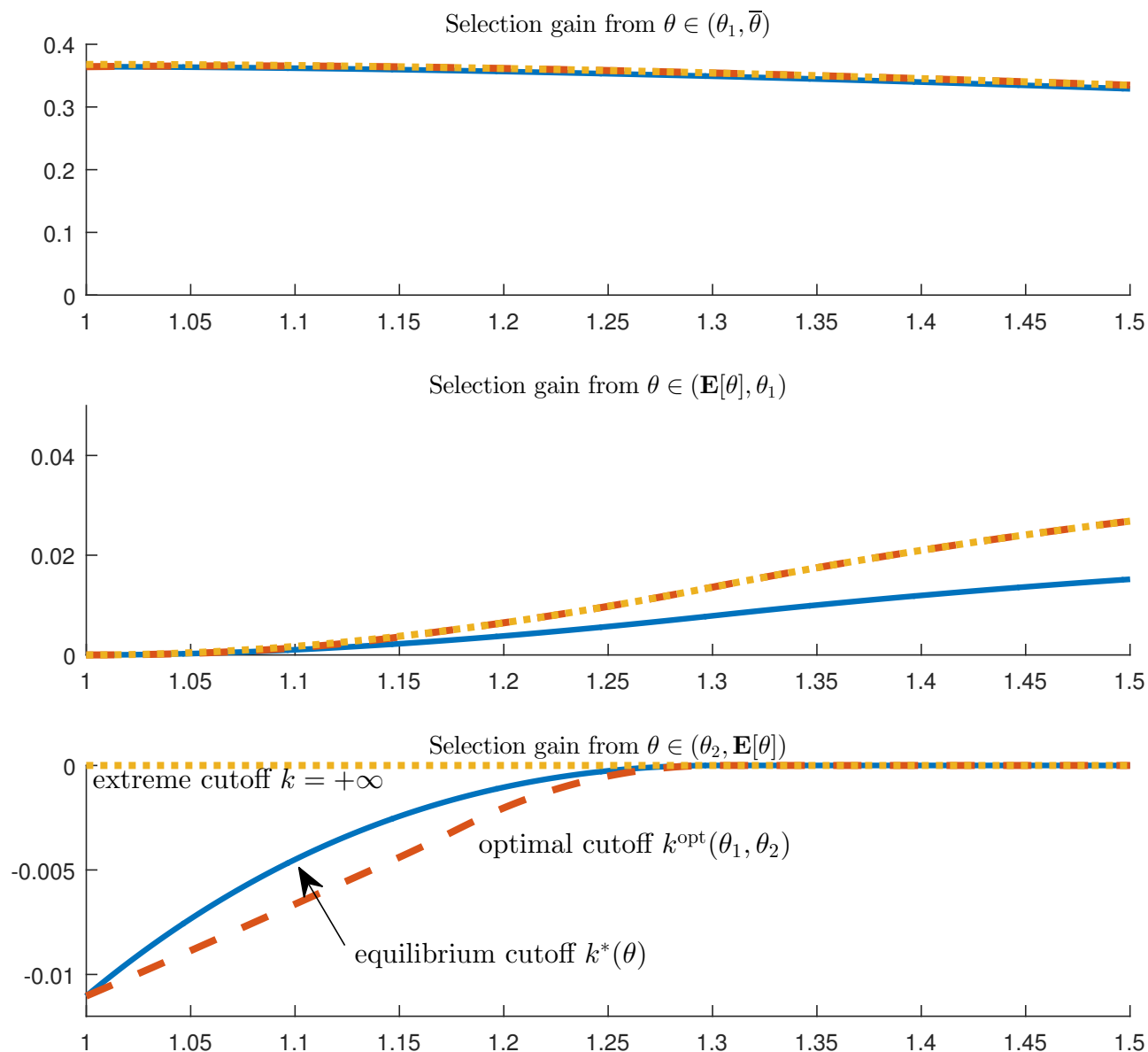


Figure 9: The contribution of groups $(\theta_1, \bar{\theta})$, $(\mathbb{E}[\theta], \theta_1)$, $(\theta_2, \mathbb{E}[\theta])$ to the normalized period-2 value of the firm. The solid curves represent the actual equilibrium values. The dashed curves represent the values with the optimal cutoff $k^{\text{opt}}(\theta_1, \theta_2)$. The dotted curves represent those with the infeasible, first-best retention policy (i.e., $k = -\infty$ for the top two groups and $k = +\infty$ for the bottom group).

types in the top group can easily pass the equilibrium cutoff, which is inflated upwards due to $c > 0$. Also, the contribution from the middle group is negligibly small even with the most favorable cutoff $k = -\infty$. This negligibility is partly because the upward bias of the equilibrium cutoff increases θ_2 and significantly lowers the probability of misreporting, as we have also seen in Figure 7.

We obtain similar results when θ is normally distributed. Figure 10 indicates that the optimal board is, once again, aggressive. In this figure, we assume that θ is normally distributed with mean 10 and standard deviation 1. All of the other parameters are the same. Although this case is more smooth than the exponential case—there is no longer a negative spike in the payoff or a discontinuity in the type distribution—we still observe similar patterns in Figure 11. The probability of misreporting is decreasing in θ_1 in the aggressive region $\theta_1 > \mathbb{E}[\theta]$ and the distortive impact of aggressiveness in the cutoff is quite limited.

These two numerical results exemplify the robustness of the results. Here, we present only two numerical results, but an aggressive board easily turns out to be optimal as long as the parameters are not too extreme. Even though analytic calculations are intractable, except for the dual uniform environment we have studied in Appendix C.2, the numerical results presented in this appendix prove how commonly aggressive boards emerge in our setting.

C.4 Additional details on continuous types

In this appendix, we present additional details regarding the analysis of the setting with continuous types. In particular, we discuss the determination of the uniform cutoff rule under exogenously specified c . The first result establishes that any retention policy must be a cutoff strategy.

Lemma 8. *Let $\hat{\theta} \in (\theta_1, \bar{\theta})$. After observing report $\hat{\theta}$, the board's optimal retention policy is a cutoff rule with threshold $k(\hat{\theta}) \in [-\infty, +\infty]$ regardless of the posterior belief $\beta_{\hat{\theta}}$.*

Observe that the cutoff $k(\hat{\theta})$ needs to be uniform across messages that the CEO uses when misreporting her type. When reporting $\hat{\theta} \neq \theta$, the manager with type θ only cares about the survival probability

$$\Pr\{\theta - d + \varepsilon > k(\hat{\theta})\} = 1 - F(k(\hat{\theta}) + d - \theta),$$

which is decreasing in $k(\hat{\theta})$. Thus, the CEO always chooses a message $\hat{\theta}$ with the lowest cutoff $k(\theta) = \inf_s k(s)$ and never uses $\hat{\theta}$ with a higher cutoff when misreporting.

We claim that all reports above θ_1 have the same cutoff level. It is still potentially possible at this stage that some or even all reports above θ_1 are out-of-equilibrium and, after such reports, the board has a very pessimistic belief and an extremely strict cutoff. We temporarily allow such implausible beliefs and cutoffs in the next lemma (Lemma 9). However, we soon claim a fuller statement (Lemma 11) after equilibrium selection (Lemma 10).

Lemma 9. *In any equilibrium with exogenous c , it occurs with probability 1 that, whenever the manager with type θ chooses a misreport $\hat{\theta} \in (\theta_1, \bar{\theta}) \setminus \{\theta\}$, the cutoff $k(\hat{\theta})$ associated with the misreport is equal to $\inf_{s \in (\theta_1, \bar{\theta})} k(s)$.*

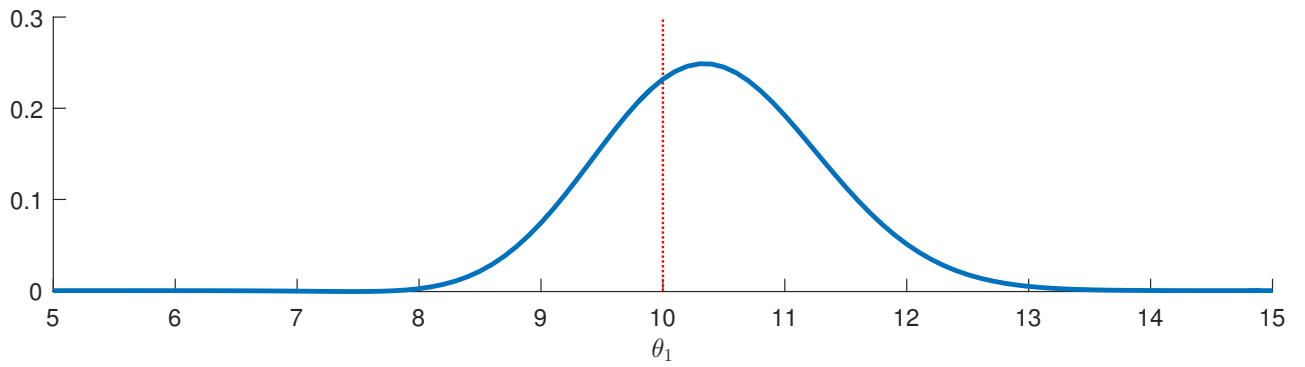


Figure 10: The objective function when $\theta \sim N(10, 1)$.

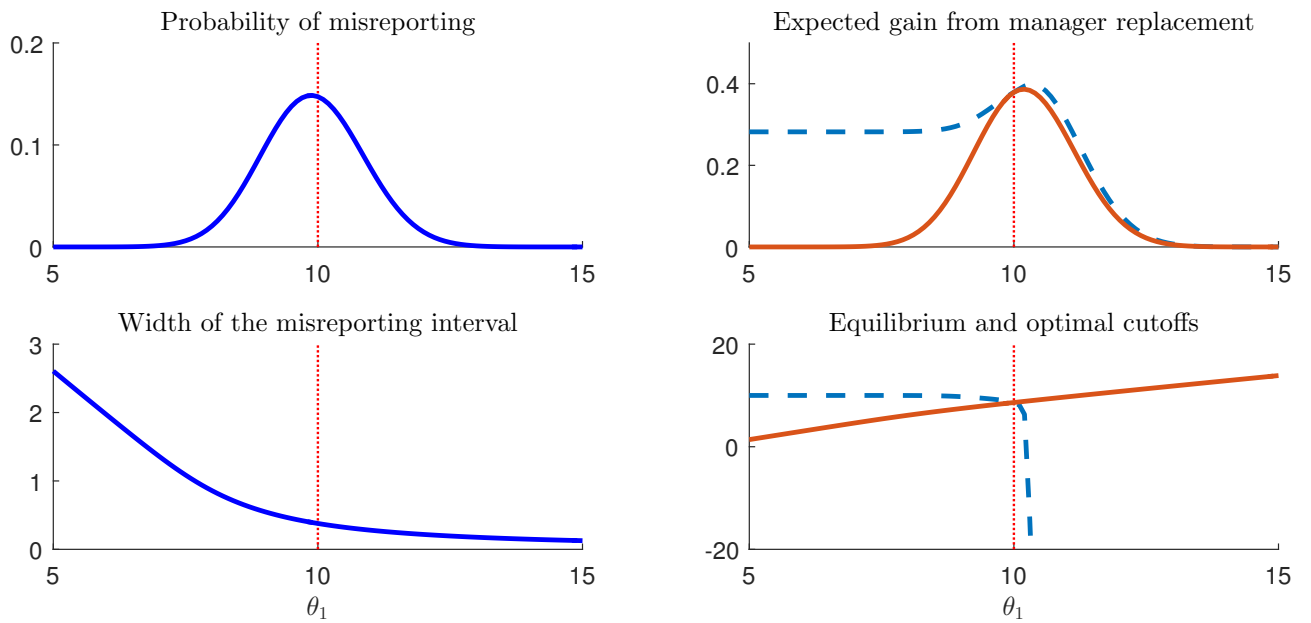


Figure 11: The counterparts of Figures 7 and 8 when $\theta \sim N(10, 1)$.

As in the case of two types, we employ the D1 criterion (see Appendix A) to eliminate the anomaly associated with out-of-equilibrium reports and to ensure truthful reporting from types $\theta > \theta_1$. As we see soon, when the CEO reports an out-of-equilibrium message $\hat{\theta} \in (\theta_1, \infty)$, the board must believe that the CEO's type is above θ_1 after we apply the D1 criterion. If this is the case, the board sets $k(\hat{\theta}) = -\infty$ (i.e., no replacement) and consequently the type $\hat{\theta}$ (as well as many other types) begins to use the message $\hat{\theta}$ to utilize the extremely friendly retention policy; consequently, message $\hat{\theta}$ is no longer out-of-equilibrium. Once out-of-equilibrium messages disappear from the interval $(\theta_1, \bar{\theta})$, all CEO types in this interval report truthfully and face the uniform cutoff $k_* = \inf_s k(s)$. (See the proof of Lemma 10 for details.)

To illustrate the D1 criterion in the present setting, consider an (ideal) equilibrium where the manager always encounters a uniform cutoff k_* after misreporting her type on the equilibrium path.⁴³ We aim to show that if $\hat{\theta} \in (\theta_1, \bar{\theta})$ is an out-of-equilibrium message, then it is type $\hat{\theta}$ that benefits the most from this message among all other types. First, observe that the manager with type θ can get at least the following payoff from misreporting:

$$U_*(\theta) = (1 - F(k_* + d - \theta))\chi + \{2\theta + \chi\},$$

in equilibrium under cutoff k_* . To guarantee this payoff or better, the out-of-equilibrium message $\hat{\theta}$ must result in a cutoff $k(\hat{\theta}) \leq k_*$ if the manager has type $\theta \neq \hat{\theta}$. When $\theta = \hat{\theta}$, the type $\hat{\theta}$ receives a much higher payoff with the truthful report $\hat{\theta}$ than the equilibrium payoff $U_*(\hat{\theta})$, as this type can uniquely boost her output y with the message $\hat{\theta}$. As a result, by the D1 criterion, the type $\hat{\theta}$ is the only type that deserves a probability weight. The actual proof is somewhat more involved than the above discussion.⁴⁴ We ultimately obtain the following result:

Lemma 10. *In any equilibrium that survives the D1 criterion, the manager with type above θ_1 truthfully reports her type.*

We now know that no report above θ_1 is an out-of-equilibrium message. In other words, we have overcome the problem of implausibly pessimistic beliefs and can strengthen the statement of Lemma 9:⁴⁵

Lemma 11. *In any equilibrium that survives the D1 criterion, the board (almost surely) employs cutoff $k_* = \min_{\hat{\theta} \in (\theta_1, \bar{\theta})} k(\hat{\theta})$ after receiving a report above θ_1 .*

⁴³This simple structure may not arise in a presumably implausible equilibrium where some types above θ_1 choose messages lower than θ_1 . We eliminate such pathological cases in the proof of Lemma 10.

⁴⁴In the derivation, we cannot assume that all misreporting types face some cutoff rule; Lemma 8 applies only to messages above θ_1 , and the other messages may induce intractable retention policies. Nevertheless, thanks to Lemma 8, at least the retention policy after $\hat{\theta}$ is tractable even though the other side—the retention policy each type faces in equilibrium—may be pathological.

⁴⁵We actually prove Lemma 11 in the proof of Lemma 10.

D Continuous retention decision

In our baseline model, the board makes a retention decision, which is naturally binary. In this appendix, we investigate how this binary specification of the board's decision contributes to this paper's results, by considering a setting where the board makes a continuous retention decision. More specifically, we consider a setting where the board makes a decision concerning the retention of the CEO but the retention decision is not binary. To make possible this paradoxical statement—whether to retain or replace the CEO is naturally binary—we assume that the board chooses $z \in [0, 1]$ as the probability of replacement, instead of directly choosing whether to retain or replace the CEO. If the payoff gain (or loss) from replacement is unchanged from $z \cdot c$, this change makes no difference from the baseline model because the newly added choices $z \in (0, 1)$ never become optimal. We hence allow the board to have a nonlinear gain (or loss) $B(z, c)$ from replacing the CEO.

In order to prevent corner solutions, we consider the case where $B(z, c)$ diverges to $-\infty$ as z goes to zero or one. Specifically, we consider the structure where $B(z, c) = z \cdot c + h(z)$ such that $h(0) = h(1) = -\infty$. To simplify the argument, $h(z)$ is a single-peaked function with $h'(0) = \infty$, $h'(1) = -\infty$, and $h''(z) < 0$ for all $z \in (0, 1)$. For example, $h(z) = \log(z - z^2)$ satisfies all of these conditions.

We derive the optimal choice of z . When the board believes the probability of the high type is μ , the decision problem of the board is to maximize

$$z\mathbb{E}[\theta] + (1 - z) [\mu\theta_H + (1 - \mu)\theta_L] + z \cdot c + h(z)$$

by controlling z . From the first-order condition $(\pi - \mu)\Delta\theta + c + h'(z) = 0$, we obtain the optimal choice of z :

$$z(\mu; c) = h'^{-1}((\mu - \pi)\Delta\theta - c).$$

Its partial derivatives are $\partial z/\partial\mu = \Delta\theta/h''(z(\mu; c)) < 0$ and $\partial z/\partial c = -1/h''(z(\mu; c)) > 0$; naturally, an improvement in the belief softens the board's retention policy and increased aggressiveness increases the probability of CEO replacement.

We then formulate the net mimicry value $V = U^m - U$ from the truth-telling payoff U and mimicry payoff U^m . Now that the board does not replace a truth-telling low-type CEO with certainty, the truth-telling payoff is not as simple as $U_{base} = \theta_L$:

$$U = \theta_L + (1 - z(0; c))\chi,$$

which is no longer independent of c . The mimicry payoff is

$$U^m = (\theta_L - d) + \chi - \chi \int_{-\infty}^{\infty} z(\mu(y, \sigma), c) f(y - \theta_L + d) dy.$$

The net mimicry value $V = U^m - U$ has a negative partial derivative with respect to σ ,

$$\frac{\partial V}{\partial \sigma} = -\chi \int_{-\infty}^{\infty} \underbrace{\frac{\partial z}{\partial \mu} \frac{\partial \mu}{\partial \sigma}}_{>0} f(y - \theta_L + d) dy < 0,$$

but the partial derivative with respect to c is ambiguous:⁴⁶

$$\frac{\partial V}{\partial c} = \underbrace{\int_{-\infty}^{\infty} \frac{\chi}{h''(z(\mu(y, \sigma); c))} f(y - \theta_L + d) dy}_{= \partial U^m / \partial c (< 0)} - \underbrace{\frac{\chi}{h''(z(0; c))}}_{= \partial U / \partial c (< 0)}.$$

By repeating the argument of Appendix 6.1, we find that we cannot determine the sign of $\partial \sigma^* / \partial c$, either.

This ambiguity emerges from the fact that the structure described in Table 1 does not hold in this setup. Because we cannot eliminate the effect of c from U , we cannot determine the sign of $\partial V / \partial c = \partial U^m / \partial c - \partial U / \partial c$ unlike in our baseline model, in which $\partial V_{base} / \partial c = \partial U_{base}^m / \partial c$ is unambiguously negative. From this argument, we learn that the binary retention decision contributes to simplify the analysis because the simple structure of Table 1 may not be achieved with a continuous decision variable, as exemplified above.

However, we note that the structure described in Table 1 fails to hold in this setup only because we forcefully eliminated the corner solution $z = 1$ by setting $B(z, c) = -\infty$ when $z = 1$. Even when the function $B(z, c)$ is quadratic (e.g., $B(z, c) = cz^2$) or concave (e.g., $B(z, c) = cz^\alpha$ with $\alpha \in (0, 1)$) in z , as long as the function $B(z, c)$ does not have an extremely negative value at $z = 1$, the corner solution $z = 1$ easily emerges after truthful reporting and the structure of Table 1 is recovered. We thus conclude that the binariness itself does not seem to be one of the crucial assumptions in the analysis of our baseline model, even though it plays a certain role to simplify the analysis through achieving the simple structure described in Table 1.

E Proofs for Appendix C

E.1 Proof of Proposition 2

When $\theta_1 \equiv \mathbb{E}[\theta] + c < \underline{\theta}$, the board unconditionally retains the CEO. In contrast, the board always replace the CEO if $\theta_1 > \bar{\theta}$. In either case, the CEO has no incentive to misreport her type and choose a truthful message as the unique optimal choice.

E.2 Proof of Proposition 3

We prove the claim in a series of lemmas.

⁴⁶We know $z(\mu(y, \sigma); c) < z(0; c)$, but this does not imply $h''(z(\mu(y, \sigma); c)) < h''(z(0; c))$ because $h''(z)$ may have a decreasing part. For example, when $h(z) = \log(z - z^2)$, $h''(z)$ is decreasing on $[1/2, 1]$ because $h'''(z) = 2[x^{-3} - (1 - x)^{-3}]$ is negative for $z > 1/2$.

E.2.1 Proof of Lemma 8

Let β denote the posterior distribution after observing $\hat{\theta}$. The expected value of type θ after observing report $\hat{\theta}$ and output y is

$$M(y) = \frac{\int_{(\underline{\theta}, \theta_1]} \theta f(y - \theta + d) d\beta + \int_{(\theta_1, \bar{\theta})} \theta f(y - \theta + d \cdot \mathbf{1}_{\{\theta \neq \hat{\theta}\}}) d\beta}{\int_{\underline{\theta}}^{\bar{\theta}} f(y - \theta + d \cdot \mathbf{1}_{\{\theta \neq \hat{\theta}\}}) d\beta}.$$

The sign of $M(y) - \theta_1$ is identical to that of

$$\begin{aligned} L(y) &= \frac{\int_{\underline{\theta}}^{\bar{\theta}} f(y - \theta + d \cdot \mathbf{1}_{\{\theta \neq \hat{\theta}\}}) d\beta}{f(y - \theta_1 + d)} \cdot (M(y) - \theta_1) \\ &= \int_{(\underline{\theta}, \theta_1]} (\theta_1 - \theta) \frac{f(y - \theta + d)}{f(y - \theta_1 + d)} d\beta - \int_{(\theta_1, \bar{\theta})} (\theta - \theta_1) \frac{f(y - \theta + d \cdot \mathbf{1}_{\{\theta \neq \hat{\theta}\}})}{f(y - \theta_1 + d)} d\beta. \end{aligned}$$

Unless the posterior β assigns probability 1 on type θ_1 , the function L is continuous and decreasing in y due to the monotone likelihood ratio property: the first integral is decreasing and the second is increasing. Therefore, it is optimal for the board to replace the manager when $y < L^{-1}(0)$ and to retain her when $y > L^{-1}(0)$. That is, a cutoff rule with $k = L^{-1}(0)$ is optimal. Here, $L^{-1}(0)$ is well-defined after continuously extending the domain of L to $[-\infty, +\infty]$.

E.2.2 Proof of Lemma 9

The manager's payoff depends only on the cutoff k when she misreports her type. Therefore, she chooses a report with the minimum cutoff and no type uses any report with a higher cutoff.

E.2.3 Proof of Lemmas 10 and 11

We repeatedly apply the following lemma in this proof.

Lemma 12. *Let $k \in [-\infty, +\infty]$ and $z : \mathbb{R} \rightarrow [0, 1]$ be a (measurable) retention policy. When $t > t'$, the following four implications hold:*

$$\begin{aligned} \int_k^\infty f(y - t') dy &\geq \int_{-\infty}^\infty z(y) f(y - t') dy \Rightarrow \int_k^\infty f(y - t) dy \geq \int_{-\infty}^\infty z(y) f(y - t) dy \\ \int_k^\infty f(y - t) dy &\leq \int_{-\infty}^\infty z(y) f(y - t) dy \Rightarrow \int_k^\infty f(y - t') dy \leq \int_{-\infty}^\infty z(y) f(y - t') dy \\ \int_{-\infty}^k f(y - t) dy &\geq \int_{-\infty}^\infty z(y) f(y - t) dy \Rightarrow \int_{-\infty}^k f(y - t') dy \geq \int_{-\infty}^\infty z(y) f(y - t') dy \\ \int_{-\infty}^k f(y - t') dy &\leq \int_{-\infty}^\infty z(y) f(y - t') dy \Rightarrow \int_{-\infty}^k f(y - t) dy \leq \int_{-\infty}^\infty z(y) f(y - t) dy. \end{aligned}$$

Moreover, the inequalities in the first two consequents are strict if $z(y) \neq \mathbf{1}_{\{y>k\}}$ on a set with a positive Lebesgue measure. The inequalities in the last two consequents are strict if $z(y) \neq \mathbf{1}_{\{y<k\}}$ on a set with a positive Lebesgue measure.

Proof. The first two implications follows from

$$\begin{aligned} & \int_k^\infty f(y-t') dy - \int_{-\infty}^\infty z(y)f(y-t') dy \\ &= f(k-t') \left\{ \int_k^\infty (1-z(y)) \frac{f(y-t')}{f(k-t')} dy - \int_{-\infty}^k z(y) \frac{f(y-t')}{f(k-t')} dy \right\} \\ &\leq f(k-t') \left\{ \int_k^\infty (1-z(y)) \frac{f(y-t)}{f(k-t)} dy - \int_{-\infty}^k z(y) \frac{f(y-t)}{f(k-t)} dy \right\} \\ &= \frac{f(k-t')}{f(k-t)} \left\{ \int_k^\infty f(y-t) dy - \int_{-\infty}^\infty z(y)f(y-t) dy \right\}. \end{aligned}$$

Here, the inequality is due to the monotone likelihood ration property. Similarly,

$$\begin{aligned} & \int_{-\infty}^k f(y-t) dy - \int_{-\infty}^\infty z(y)f(y-t) dy \\ &\geq \frac{f(k-t)}{f(k-t')} \left\{ \int_{-\infty}^k f(y-t') dy - \int_{-\infty}^\infty z(y)f(y-t') dy \right\}. \end{aligned}$$

proves the latter half. In either case, the inequality is strict when the condition in the statement is satisfied. \square

The following lemma constitutes an essential part of this proof.

Lemma 13. *Consider an equilibrium that survives the D1 criterion. If message $\theta \in (\theta_1, \bar{\theta})$ is out of equilibrium, then the board assigns probability 1 on types above θ_1 after this message. Consequently, no message above θ_1 is out of equilibrium.*

Proof. Let $\hat{\theta}$ be a message that type θ uses in equilibrium and let $\hat{z}(y)$ be the equilibrium retention policy for message $\hat{\theta}$.

We first prune the possibility that type $\theta' \in (\underline{\theta}, \theta_1] \setminus \{\hat{\theta}\}$ chooses message θ . Suppose that the type θ' weakly prefers message θ to $\hat{\theta}$ if the board uses a cutoff rule with cutoff k for message θ . Since type θ' is neither θ nor $\hat{\theta}$, we have

$$\int_k^\infty f(y-(\theta'-d)) dy \geq \int_{-\infty}^\infty \hat{z}(y)f(y-(\theta'-d)) dy$$

and thus, by Lemma 12,

$$\int_k^\infty f(y-\theta) dy \geq \int_{-\infty}^\infty \hat{z}(y)f(y-\theta) dy.$$

That is, type θ has a higher retention rate with the truthful message θ than the equilibrium message $\hat{\theta}$. Since type θ gains d in addition by truth-telling, this type strictly prefers the truthful message. Therefore, after observing message θ , the board assigns no probability on the set $(\underline{\theta}, \theta_1] \setminus \{\hat{\theta}\}$.

We then consider type $\hat{\theta}$. Once again suppose the board uses cutoff k for message θ . The type $\hat{\theta}$ weakly prefers message θ to $\hat{\theta}$ only if

$$\int_k^\infty f(y - (\hat{\theta} - d)) dy > \int_{-\infty}^\infty \hat{h}(y) f(y - \hat{\theta}) dy. \quad (\text{E.1})$$

First suppose $\theta - d \geq \hat{\theta}$. In this case, define \hat{k} by

$$\int_{-\infty}^\infty \hat{z}(y) f(y - \hat{\theta}) dy = \int_{-\infty}^{\hat{k}} f(y - \hat{\theta}) dy. \quad (\text{E.2})$$

By combining (E.1) and (E.2), we obtain $F(\hat{\theta} - d - k) > F(\hat{k} - \hat{\theta})$, or equivalently, $\hat{k} < 2\hat{\theta} - d - k$. Also, from (E.2), by Lemma 12,

$$\begin{aligned} \int_{-\infty}^\infty \hat{z}(y) f(y - (\theta - d)) dy &\leq \int_{-\infty}^{\hat{k}} f(y - (\theta - d)) dy = F(\hat{k} - \theta + d) \\ &< F(2\hat{\theta} - \theta - k) < F(\theta - k) = \int_k^\infty f(y - \theta) dy. \end{aligned}$$

That is, type θ have a higher retention rate with the truthful message than message $\hat{\theta}$. Therefore, in this case, we prune the possibility that type $\hat{\theta}$ chooses message θ .

Now suppose $\theta - d < \hat{\theta}$. This time, we define \hat{k} by

$$\int_{-\infty}^\infty \hat{h}(y) f(y - \hat{\theta}) dy = \int_{\hat{k}}^\infty f(y - \hat{\theta}) dy. \quad (\text{E.3})$$

From (E.1) and (E.3), we have $\hat{\theta} - d - k > \hat{\theta} - \hat{k}$ and thus $\hat{k} > k + d$. By Lemma 12, equation (E.3) implies

$$\begin{aligned} \int_{-\infty}^\infty \hat{z}(y) f(y - (\hat{\theta} - d)) dy &\leq \int_{\hat{k}}^\infty f(y - (\hat{\theta} - d)) dy = F(\hat{\theta} - d - \hat{k}) \\ &< F(\hat{\theta} - d - (k - d)) < F(\theta - k) = \int_k^\infty f(y - \theta) dy. \end{aligned}$$

Hence, again, type θ strictly prefers the truthful message to the equilibrium message. In either case, the board assigns no probability on $(\underline{\theta}, \theta_1]$ after observing message θ in any D1 equilibrium.

To show the second part of the statement, suppose $\hat{\theta}$ is out of equilibrium. By the first part of this lemma, the board optimally retains the manager for sure after observing that message. If this is the case, the type $\hat{\theta}$ should choose the truthful message $\hat{\theta}$, which contradicts the assumption that message $\hat{\theta}$ is out of equilibrium. \square

We first show Lemma 11 by combining Lemmas 9 and 13.

Proof of Lemma 11

Let $k(\hat{\theta})$ denote the cutoff for message $\hat{\theta} \in (\theta_1, \bar{\theta})$. Suppose to the contrary $k(\hat{\theta}) > k(\hat{\theta}')$ for some $\hat{\theta}, \hat{\theta}' \in (\theta_1, \bar{\theta})$. Then, no type uses message $\hat{\theta}$ as a misreport by Lemma 9. By Lemma 9, the message $\hat{\theta}$ needs to be used by type $\hat{\theta}$. In this case, the board retains the manager for sure, i.e., $k(\hat{\theta}) = -\infty$. This contradicts with $k(\hat{\theta}) > k(\hat{\theta}')$.

Proof of Lemma 10

First observe that, by Lemma 11, the type θ^* prefers the truthful message to any other message above θ_1 because all of these messages use the same cutoff k_* . We show that the retention rate for any message $\hat{\theta} \in (\underline{\theta}, \theta_1]$ does not exceed the truthful counterpart. That is,

$$\int_{-\infty}^{\infty} \hat{z}(y) f(y - \hat{\theta}^* - d) dy \leq \int_{k_*}^{\infty} f(y - \theta^*) dy, \quad (\text{E.4})$$

where $\hat{z}(y)$ is the retention policy for message $\hat{\theta}$. If this is the case, the message θ^* is the unique optimal choice for the type θ^* .

The condition (E.4) is clearly satisfied when $k_* = -\infty$. We thus assume $k_* > -\infty$. In this case, there must be a set of types with positive Lebesgue measure that report some message above θ_1 with positive probability because otherwise the board assigns probability 1 on $(\theta_1, \bar{\theta})$ and sets $k(\theta) = -\infty$ for some message θ above θ_1 . At least one of such types differs from $\hat{\theta}$ and let θ_* denote this type. Since type θ_* weakly prefers the cutoff k_* to the retention policy for $\hat{\theta}$,

$$\int_{-\infty}^{\infty} \hat{z}(y) f(y - (\theta_* - d)) dy \leq \int_{k_*}^{\infty} f(y - (\theta_* - d)) dy. \quad (\text{E.5})$$

By Lemma 12, the inequality (E.5) implies

$$\int_{-\infty}^{\infty} \hat{z}(y) f(y - (\theta^* - d)) dy \leq \int_{k_*}^{\infty} f(y - (\theta^* - d)) dy = F(\theta^* - d - k_*) \leq F(\theta^* - k_*).$$

This is the condition (E.4) and thus the truthful message is uniquely optimal. Therefore, any type above θ_1 reports the truthful message with probability 1.

E.3 Proof of Lemma 6

First note that, on the equilibrium path, the CEO is replaced if her type is below θ_1 and her message is θ_1 or below because all the types above θ_1 report truthful messages (Lemma 10).⁴⁷ Thus, for the types below θ_1 , it is optimal to choose either (a) truthful messages or

⁴⁷The entire proof should be interpreted as a measure theoretic statements. For example, we allow some types below θ_1 is retained even with messages below θ_1 as long as the set of such types has Lebesgue measure 0.

(b) messages above θ_1 accompanied with the uniform cutoff. The threshold θ_2 is defined, by (eq:theta2), as the type that makes the CEO indifferent between these two choices. As explained in the text, types more than the threshold θ_2 have a higher chance of survival than the indifferent type θ_2 and thus prefer (b); types below θ_2 prefer (a). Therefore, in equilibrium, types below θ_2 (and below θ_1) report truthful messages, whereas types above θ_2 (but below θ_1) choose messages above θ_1 .

E.4 Proof of Lemma 7

We first show the condition (C.3) is necessary when $\theta_2 \in (-\infty, \theta_1)$. Since the board needs to uniformly choose a single cutoff k_* for almost every messages $\hat{\theta}$ above θ_1 , the uniform cutoff k_* satisfies the first-order condition

$$\mathbb{E} \left[(\theta - \theta_1) f(k_* + d \cdot \mathbf{1}_{\{\theta \neq \hat{\theta}\}} - \theta) \mid \hat{\theta} \right] = 0$$

for such messages. By the law of total expectation,

$$\begin{aligned} 0 &= \mathbb{E} \left[\mathbf{1}_{\{\hat{\theta} > \theta_1\}} \cdot (\theta - \theta_1) f(k_* + d \cdot \mathbf{1}_{\{\theta \neq \hat{\theta}\}} - \theta) \right] \\ &= \mathbb{E} \left[\mathbf{1}_{\{\theta > \theta_1\}} \cdot (\theta - \theta_1) f(k_* - \theta) \right] - \mathbb{E} \left[\mathbf{1}_{\{\theta \in (\theta_1, \theta_2)\}} \cdot (\theta - \theta_1) f(k_* + d - \theta) \right]. \end{aligned}$$

The last two expectations represent the left-hand and right-hand sides of the desired condition (C.3).

The condition (C.3) is equivalent to $J(k_*; \theta_2) = 0$, where

$$J(k_*; \theta_2) = \int_{\theta_1}^{\bar{\theta}} (\theta - \theta_1) \frac{f(k_* - \theta)}{f(k_* - \theta_1)} g(\theta) d\theta - \int_{\theta_2}^{\theta_1} (\theta_1 - \theta) \frac{f(k_* + d - \theta)}{f(k_* - \theta_1)} g(\theta) d\theta. \quad (\text{E.6})$$

By the monotone likelihood ratio property, the condition $J(k_*; \theta_2) = 0$ has a unique solution $k_*(\theta_2)$ given θ_2 . The solution is decreasing in θ_2 because $J(k_*; \theta_2)$ is increasing in k_* and non-decreasing in θ_2 . Also, $k_*(\theta_2)$ is a continuous function because $J(k_*; \theta_2)$ is jointly continuous (and increasing in k_*). As θ_2 approaches θ_1 , the solution $k_*(\theta_2)$ decreases to $-\infty$ because $J(k_*; \theta_2)$ converges to a positive value as $\theta_2 \nearrow \theta_1$, whenever cutoff k_* is finite.

E.5 Proof of Theorem 5

Let (θ_2^*, k^*) be the unique fixed point. We first construct an equilibrium that survives the D2 (and thus D1) criteria.

Existence

We start by specifying how misreporting types mix their reports. Define $h(\hat{\theta})$ for each $\hat{\theta} \in (\theta_1, \bar{\theta})$ by

$$h(\hat{\theta}) = \frac{(\hat{\theta} - \theta_1)f(k^* - \hat{\theta})g(\hat{\theta})}{\int_{\theta_2^*}^{\theta_1} (\theta_1 - \theta)f(k^* + d - \theta)g(\theta) d\theta}.$$

This function h works as a density function:

$$\int_{\theta_1}^{\bar{\theta}} h(\hat{\theta}) d\hat{\theta} = \frac{\int_{\theta_1}^{\bar{\theta}} (\hat{\theta} - \theta_1)f(k^* - \hat{\theta})g(\hat{\theta}) d\hat{\theta}}{\int_{\theta_2^*}^{\theta_1} (\theta_1 - \theta)f(k^* + d - \theta)g(\theta) d\theta}$$

is equal to 1 because k^* satisfies the condition (C.3).

We consider the following strategies and beliefs. The manager with type $\theta \in [\theta_2^*, \theta_1]$ uses the density $h(\hat{\theta})$ to randomize the messages $\hat{\theta} \in (\theta_1, \bar{\theta})$. The manager with some other type reports a truthful message. After observing message $\hat{\theta} \in (\theta_1, \bar{\theta})$, the board calculates a posterior belief by the Bayes' rule and employs the uniform cutoff k^* . After message $\hat{\theta} \in (\underline{\theta}, \theta_1]$, the board sets cutoff $k = +\infty$ (i.e., replacement for sure).

We briefly discuss the optimality of the strategies. The optimality of k^* follows from the fact that the density function $h(\hat{\theta})$ satisfies the first-order condition (C.2) for all $\hat{\theta} \in (\theta_1, \bar{\theta})$. The extreme cutoff $k = +\infty$ is a best response for the lower messages because after these messages the board assigns no probability on types above θ_1 . The manager with type above θ_1 chooses truthful messages as a unique optimum to get the highest retention rate and the additional productivity d . By the definition of θ_2 , the remaining types also choose optimal messages for them.

It remains to show that this belief systems survives the D1 and D2 criteria. Consider a cutoff $k_\varepsilon = k^* - \varepsilon$ slightly lower than k^* . Suppose that this cutoff k_ε is accompanied with an out-of-equilibrium message $\hat{\theta} \in [\theta_2, \theta_1]$. Then, when ε is sufficiently small, type $\theta = \hat{\theta}$ strictly prefer the truthful message $\hat{\theta}$ for this type to the equilibrium misreporting, whereas all the other types get worse off with this message than their equilibrium messages. Note that the cutoff k_ε becomes a best response for the board by controlling the belief for this message; when the board assigns probability p on $(\bar{\theta} + \theta_1)/2$ and $1 - p$ on $(\underline{\theta} + \theta_1)/2$, we can make any level of cutoff a best response by correctly adjusting p . Therefore, neither the D1 or D2 criterion can eliminate such a belief.

D2 Criterion

The necessary conditions for D1 equilibria are already shown by Lemmas 10–6. It remains to show that, in any D2 equilibrium, the board must replace the CEO after (almost) every message $\hat{\theta} \in (\theta_2, \theta_1)$.

We claim that the board never assign probability on types above $\hat{\theta}$ for all out-of-equilibrium messages $\hat{\theta}$. To this end, we assume that the manager with type $\theta^* > \hat{\theta}$ weakly prefers the message $\hat{\theta}$ with a retention policy z to the equilibrium message for type θ^* and show that some type θ_z strictly prefers the message $\hat{\theta}$ to the equilibrium message for type θ_z .

First consider the case that the retention policy z differs from the cutoff rule with cutoff k^* in a measure-theoretic sense; i.e., $\{y : z(y) \neq \mathbf{1}_{\{y > k^*\}}\}$ has a positive Lebesgue measure. The manager with type $\theta^* > \hat{\theta}$ weakly prefers the message $\hat{\theta}$ with the above retention policy to the equilibrium message only if the retention rate with message $\hat{\theta}$ and retention policy z is at least as high as with some misreport above θ_1 and cutoff k^* :

$$\int_{-\infty}^{\infty} z(y)f(y - (\theta^* - d)) dy \geq \int_{k^*}^{\infty} f(y - (\theta^* - d)) dy.$$

Let $\theta_* \in (\theta_2, \hat{\theta})$ be a type that chooses a message above θ_1 and faces cutoff k^* in equilibrium. By Lemma 12, the type θ_* has a higher chance of retention with message $\hat{\theta}$ and retention policy z than with the equilibrium message and the cutoff k^* . That is, when such retention policy z is accompanied with message $\hat{\theta}$, type θ_* strictly prefers the message $\hat{\theta}$ to the equilibrium message for this type whenever type θ^* weakly prefers the message $\hat{\theta}$ to the equilibrium message for type θ .

Now consider the case that the retention policy z is identical to the cutoff rule with k^* . (We need the D2 criterion, instead of D1, just for this part.)⁴⁸ In this case, the type $\hat{\theta}$ prefers the truthful message $\hat{\theta}$ to its equilibrium choice, accompanied with the uniform cutoff k^* , whereas the type θ is indifferent between $\hat{\theta}$ and the equilibrium choice (or prefers the latter).⁴⁹ That is, even when z is identical to the cutoff rule with k^* , we can find a type (in this case, type $\hat{\theta}$) strictly prefer message $\hat{\theta}$ to the equilibrium message.

We have shown that whenever type $\theta^* > \hat{\theta}$ weakly prefers message $\hat{\theta}$ to its equilibrium message, there exists some type that strictly prefers message $\hat{\theta}$ to the equilibrium message for that type. Therefore, after observing $\hat{\theta}$, the board assigns probability 1 on types below $\hat{\theta}$ ($< \theta_1$) and replaces the CEO regardless of the realization of y .

E.6 Proof of Proposition 5

Case (iv) is obvious. We focus on cases (i)–(iii); i.e., $|c| < \Delta/2$. We first provide a necessary condition for equilibrium. Let $\theta_* = \max\{\underline{\theta}, \theta_2\}$ and $\theta^* = \min\{\bar{\theta}, k + q\}$.

Lemma 14. *Suppose $|c| < \Delta/2$. In equilibrium, $\theta_1 - \theta_* = \theta^* - \theta_1$ and $k \in [\theta_1 - q, \theta_* - d + q]$.*

Proof. Given $\theta_2 < \theta_1$, the objective of the board is to maximize

$$B(k; \theta_2) = \int_{\theta_1}^{\bar{\theta}} (\theta - \theta_1)F(k - \theta) d\theta + \int_{\theta_*}^{\theta_1} (\theta - \theta_1)F(k + d - \theta) d\theta. \quad (\text{E.7})$$

⁴⁸The type $\hat{\theta}$ does not necessarily eliminate all the other types through the D1 criterion. Consider a type $\theta \in (\theta_2, \hat{\theta} + d/2)$. We can easily show that when the board uses a reversed cutoff rule with a cutoff level k that makes type θ indifferent (i.e., $F(k - \theta) = F(\theta - k^*)$), the retention rate for type $\hat{\theta}$ is lower with the truthful message $\hat{\theta}$ than with the equilibrium misreports (i.e., $F(k - \hat{\theta} - d) < F(\theta - \hat{\theta} - d)$). In particular, the type θ can be more than θ_1 when $\hat{\theta}$ is close enough to θ_1 ; that is, the D1 criterion may not be able to eliminate some types above θ_1 .

⁴⁹The type θ may get a better deal than cutoff k^* in equilibrium because the posterior belief is indeterminate on a set of measure 0. If it is the case, we do not need the argument for the D2 criterion; the D1 criterion suffices. In general, of course, we need the D2 criterion to obtain the desired result.

The function $B(k; \theta_2)$ is zero when no type survives (i.e., $k \geq \bar{\theta} + q$). Also, $B(k; \theta_2)$ is positive when $k \in [\theta_1 - d + q, \bar{\theta} + q)$ because no type below θ_1 survives. Among these values of k , the lowest value $k = \theta_1 - d + q$ gives the highest rate of retention and the highest value of $B(k)$ within the interval. Thus, the values of $k > \theta_1 - d + q$ are all suboptimal. Similarly, it is suboptimal to choose $k < \theta_1 - q$ because the retention rate of types below θ_1 increases as k decreases on that region. Therefore, we can focus on $k \in I = [\theta_1 - q, \theta_1 - d + q]$.

We further partition the interval I into three intervals: $I_1 = [\theta_1 - q, \bar{\theta} - q]$, $I_2 = (\bar{\theta} - q, \theta_* - d + q)$, and $I_3 = [\theta_* - d + q, \theta_1 - d + q]$. The second interval I_2 is nonempty because of the second regularity condition (C.5). Actually, $k \in I_3$ never occurs in equilibrium because $k \in I_3$ implies that the threshold type θ_2 has no chance of survival. This contradicts the definition of θ_2 : the manager with type θ_2 needs to be indifferent between truth-telling and misreporting.

We investigate the optimality condition for k . If $k \in I_1$, then $\theta^* = k + q$ and the function $B(k; \theta_2)$ becomes

$$B(k; \theta_2) = \int_{\theta^*}^{\bar{\theta}} (\theta - \theta_1) \cdot 1 \, d\theta + \int_{\theta_1}^{\theta^*} (\theta - \theta_1) \frac{k - \theta + q}{2q} \, d\theta + \int_{\theta_*}^{\theta_1} (\theta - \theta_1) \frac{k + d - \theta + q}{2q} \, d\theta. \quad (\text{E.8})$$

On the second interval I_2 , we obtain $\theta^* = \bar{\theta}$ and

$$B(k; \theta_2) = \int_{\theta_1}^{\theta^*} (\theta - \theta_1) \frac{k - \theta + q}{2q} \, d\theta + \int_{\theta_*}^{\theta_1} (\theta - \theta_1) \frac{k + d - \theta + q}{2q} \, d\theta, \quad (\text{E.9})$$

and (E.10). In either case,

$$\frac{\partial B}{\partial k} = \frac{1}{4q} \left\{ (\theta_1 - \theta_*)^2 - (\theta^* - \theta_1)^2 \right\} \quad (\text{E.10})$$

and thus in equilibrium $(\theta_1 - \theta_*)^2 = (\theta^* - \theta_1)^2$, or equivalently, $\theta_1 - \theta_* = \theta^* - \theta_1$ must be satisfied. \square

Consider case (i) of this proposition. By Lemma 14, $\theta_* = \theta_2 > \underline{\theta}$ must hold because otherwise $\theta^* - \theta_1 < \Delta/2 < \theta_1 - \underline{\theta} = \theta_1 - \theta_*$. Also, $\theta^* = \bar{\theta} < k + q$ because the second regularity condition (C.5) implies

$$\underline{\theta} - \theta_{2,0} > \bar{\theta} - q \quad (\text{E.11})$$

and thus $k + q \geq \underline{\theta} + q - \theta_{2,0} > \bar{\theta}$. Therefore, an equilibrium candidate is uniquely given by the equilibrium condition $\bar{\theta} - \theta_1 = \theta_1 - \theta_2$. To verify it is indeed an equilibrium, we simply need to confirm $\theta_2 = 2\theta_1 - \bar{\theta} > \underline{\theta}$ (by $c > 0$) and $k = 2\theta_1 - \bar{\theta} - \theta_{2,0} > \bar{\theta} - q$ (by (E.11)).

We next consider case (ii). In this case, $\theta^* = k - q < \bar{\theta}$ holds because otherwise $\theta - \theta_1 > \Delta/2 > \theta_1 - \theta_*$. Once again by (E.11), we obtain $\theta_2 = k + \theta_{2,0} < \bar{\theta} + q + \theta_{2,0} < \underline{\theta}$ and thus $\theta_* = \underline{\theta}$. These conditions uniquely determine an equilibrium: $k = 2\theta_1 - \underline{\theta} - q$ and $\theta_2 = 2\theta_1 - \underline{\theta} - q + \theta_{2,0}$.

In case (iii), $\theta^* = \bar{\theta}$ and $\theta_* = \underline{\theta}$ hold. Suppose otherwise. Then, $\theta^* - \theta_1 = \theta_1 - \theta_* < \Delta/2$ and thus $\theta^* = k + q$ and $\theta_* = \theta_2$ hold. However, $\theta_2 = k + \theta_{2,0} < \bar{\theta} - q + \theta_{2,0} < \underline{\theta}$ by (E.11); a contradiction. In this case, any value of k works as an equilibrium cutoff as long as $k + q \geq \bar{\theta}$ and $k + \theta_{2,0} \leq \underline{\theta}$.

E.7 Proof of Theorem 6

We first calculate the payoff function for shareholders.

Lemma 15. *Assume the regularity conditions (C.5) and (C.4). The equilibrium payoff for the shareholder with exogenous c is*

$$V_A(c) = \frac{1}{\Delta q} \left\{ \frac{2}{3} \cdot c^3 - \left(\frac{dA}{4} + \frac{\Delta}{2} \right) \cdot c^2 + d \left(\alpha \cdot \frac{\Delta}{2} + q \right) \cdot c + v_0 \right\}$$

when $c \in (0, \Delta/2)$, and

$$V_F(c) = \frac{1}{\Delta q} \left\{ -\frac{2}{3} \cdot c^3 - \left(\frac{d}{4} + \frac{\Delta}{2} \right) \cdot c^2 - dq \cdot c + v_0 \right\}$$

when $c \in (-\Delta/2, 0)$, where $v_0 = [2\Delta^2 - 3d(8q - \Delta)]\Delta/48$.

Proof. First assume $c \in (0, \Delta/2)$. By Proposition 5, we have $k = 2\theta_1 - \bar{\theta} - \theta_{2,0}$, $k < \bar{\theta} - q$, and $\theta_2 = 2\theta_1 - \bar{\theta}$ in equilibrium. Thus, the payoff for shareholders, multiplied by Δq , is

$$\begin{aligned} \Delta q V_A &= q \left\{ -d(\theta_1 - \theta_2) + \int_{\theta_1}^{\bar{\theta}} (\theta - \mu) F(\theta - k) d\theta + \int_{\theta_2}^{\theta_1} (\theta - \mu) F(\theta - d - k) d\theta \right\} \\ &= -dq \left(\frac{\Delta}{2} - c \right) + \frac{1}{2} \left[\mu(k - q)\theta - (k - q + \mu) \frac{\theta^2}{2} + \frac{\theta^3}{3} \right]_{\theta=\mu+c}^{\mu+\Delta/2} \\ &\quad + \frac{1}{2} \left[\mu(k + d - q)\theta - (k + d - q + \mu) \frac{\theta^2}{2} + \frac{\theta^3}{3} \right]_{\theta=\mu+2c-\Delta/2}^{\mu+c} \\ &= -dq \left(\frac{\Delta}{2} - c \right) + \left[\frac{c^3}{3} - \frac{2d(1 + \alpha) - \Delta}{8} c^2 - \frac{\Delta^2}{8} c + \frac{\Delta^2}{96} \{6d(1 + \alpha)d + 5\Delta\} \right] \\ &\quad + \left[\frac{c^3}{3} - \frac{6d\alpha + 3\Delta}{8} c^2 + \frac{\Delta}{8} (4d\alpha + \Delta)c - \frac{\Delta^2}{96} (6d\alpha + \Delta) \right]. \end{aligned}$$

We obtain the desired expression by simplifying the above expression.

We next consider the case of $c \in (-\Delta/2, 0)$. We know that $k = 2\theta_1 - q - \underline{\theta}$, $k < \bar{\theta} - q$

and $\theta_2 < \underline{\theta}$ due to Proposition 5. Hence,

$$\begin{aligned} \Delta q V_F &= q \left\{ -d(\theta_1 - \theta_2) + \int_{k+q}^{\bar{\theta}} (\theta - \mu) d\theta + \int_{\theta_1}^{k+q} (\theta - \mu) F(\theta - k) d\theta + \int_{\underline{\theta}}^{\theta_1} (\theta - \mu) F(\theta - d - k) d\theta \right\} \\ &= -dq \left(c + \frac{\Delta}{2} \right) - q [2c^2 + \Delta c] + \left[-\frac{c^3}{3} + \frac{3}{8}(4q - \Delta)c^2 + \left(\Delta q - \frac{\Delta^2}{8} \right) c + \frac{\Delta^2}{96}(12q - \Delta) \right] \\ &\quad + \left[-\frac{c^3}{3} + \frac{1}{8}(4q - \Delta - 2d)c^2 + \frac{\Delta^2}{8} \cdot c + \frac{\Delta^2}{96}(6d + 5\Delta - 12q) \right]. \end{aligned}$$

Once again, we obtain the desired expression after a few more calculations. \square

These two cubic functions V_A and V_F may have a local maximum at c_A^* and c_F^* , respectively. We later see that the locally maximum values (i.e., $V_A(c_A^*)$ and $V_F(c_F^*)$) are proportional to

$$\begin{aligned} \Omega(x) &= -2(1 + 4x)^3 d^3 + 2(1 + 4x)d^2 \left[(1 + 4x)\sqrt{D(x)} - 6\Delta + 48q \right] \\ &\quad + 8d(\Delta - 8q) \left[\sqrt{D(x)} + 3\Delta \right] + 8\Delta^2 \left[\sqrt{D(x)} + 2\Delta \right] \end{aligned}$$

when $x = \alpha$ and 0, respectively, where $D(x) = d^2(1 + 4x)^2 + 4d\Delta - 32dq + 4\Delta^2$. Note that $D(\alpha)$ is positive because

$$D(\alpha) > 16\alpha^2 d^2 + 8\alpha d^2 + d^2 + \frac{16dq}{2\alpha + 1} - 32dq + \frac{64q^2}{(2\alpha + 1)^2} = \frac{(8\alpha^2 d + 6\alpha d + d - 8q)^2}{(2\alpha + 1)^2} > 0. \quad (\text{E.12})$$

The inequality follows from the first regularity condition (C.4), or equivalently, $\Delta > 4q/(1 + 2\alpha)$.

Lemma 16. *If $D(0) > 0$ and (C.5) holds, then $\Omega'(x) > 0$ for all $x \in (0, \alpha)$. Also, $\Omega'(x) > 0$ for all positive $x > \frac{4q - \Delta}{2\Delta}$.*

Proof. The first-order derivative of Ω is given by

$$\frac{1}{24d^2} \Omega'(x) = 16q - 2\Delta - d(1 + 4x)^2 + (1 + 4x)\sqrt{D(x)}. \quad (\text{E.13})$$

If $16q \geq 2\Delta + d(1 + 4x)^2$, then (E.13) is positive; this is the case when (C.5) holds. Suppose otherwise. Then, (E.13) is positive if and only if

$$\lambda(x) = \frac{1}{32} \left\{ (1 + 4x)^2 D(x) - [d(1 + 4x)^2 + 2\Delta - 16q]^2 \right\} = x(1 + 2x)\Delta^2 - 2q(4q - \Delta)$$

is positive. Since

$$\lambda(x) > \lambda\left(\frac{4q - \Delta}{2\Delta}\right) = 4q\left(\frac{4q - \Delta}{2}\right) - 2q(4q - \Delta) = 0,$$

we obtain $\Omega'(x) > 0$. □

Lemma 17. *Assume the regularity conditions (C.5) and (C.4). The function V_A is uniquely maximized at $c_A^* = (dA + 2\Delta - \sqrt{D(\alpha)})/8$ within the domain $(0, \Delta/2)$. The maximum value is positive.*

Proof. The first-order derivative of V_A has two roots at $c = (dA + 2\Delta \pm \sqrt{D(\alpha)})/8$. We show the smaller root $c_A^* = (dA + 2\Delta - \sqrt{D(\alpha)})/8$ is a unique local maximizer on the domain $(0, \Delta/2)$. The smaller root is a unique local minimizer on the unrestricted domain \mathbb{R} as an elementary property of cubic functions. Thus, we only need to show $c_A^* \in (0, \Delta/2)$. The root c^* is always positive because

$$D(\alpha) = (dA + 2\Delta)^2 - 16d(\alpha\Delta + 2q) < (dA + 2\Delta)^2.$$

We show that the condition (C.4) guarantees $c_A^* < \Delta/2$, or equivalently, $\sqrt{D(\alpha)} > 2\Delta - dA$. Since (C.4) is equivalent to $q < \Delta(1 + 2\alpha)/4$, we obtain

$$D > d^2 A^2 + 4d\Delta - 32d \cdot \frac{\Delta(1 + 2\alpha)}{4} + 4\Delta^2 = (2\Delta - dA)^2.$$

Hence, $\sqrt{D(\alpha)} > \sqrt{(2\Delta - dA)^2} = 2\Delta - dA$ if $2\Delta - dA \geq 0$; otherwise, $\sqrt{D(\alpha)} > 0 > 2\Delta - dA$. In either case, we obtain $c^* < \Delta/2$ and thus the point c_A^* is a local maximizer on the domain $(0, \Delta/2)$.

Now we show $V_A(c_A^*)$ is always positive. After several calculations, we obtain $V_A(c_A^*) = \frac{1}{768\Delta q}\Omega(\alpha)$. By Lemma 16,

$$\begin{aligned} 768\Delta q V_A(c_A^*) &= \Omega(\alpha) > \Omega\left(\frac{4q - \Delta}{2\Delta}\right) \\ &= 2\Delta^{-3} [2\Delta^2 - d(8q - \Delta)]^2 \left\{ |2\Delta^2 - d(8q - \Delta)| + [2\Delta^2 - d(8q - \Delta)] \right\} \geq 0. \end{aligned}$$

because $\alpha > \frac{4q - \Delta}{2\Delta}$ by (C.4).

Lastly, observe that the local maximum attained at c_A^* is indeed a global maximum on $(0, \Delta/2)$ because $V_A'(0) > 0$ and $V_A(\Delta/2) = 0$. □

To complete the proof of Theorem 6, we show V_F never exceeds $V_A(c^*)$. First note if V_F does not have a local maximizer on its domain $(-\Delta/2, 0)$, then V_F is negative because V_F is a cubic function with $V_F'(0) < 0$ and a negative coefficient on c^3 . Suppose a local maximizer c_F^* exists. Then, it must be the larger root of $V_F'(c) = 0$; that is, $c_F^* = (\sqrt{D(0)} - d - 2\Delta)/8$. The local maximum is given by $V_F(c_F^*) = \frac{1}{768\Delta q}\Omega(0)$, which is less than $V_A(c_A^*) = \frac{1}{768\Delta q}\Omega(\alpha)$ by Lemma 16 and the second regularity condition (C.5). In either case, V_F cannot exceed $V_A(c_A^*)$ and it is therefore optimal for shareholders to choose $c = c_A^*$.

E.8 Proof of Proposition 6

The first-order derivatives of c_A^* are

$$\begin{aligned}\frac{\partial c_A^*}{\partial d} &= \frac{1}{8\sqrt{D}} \left\{ (1+4\alpha)\sqrt{D} + [16q - d(1+4\alpha)^2 - 2\Delta] \right\} \\ \frac{\partial c_A^*}{\partial \Delta} &= \frac{1}{4\sqrt{D}} \left\{ \sqrt{D} - (d+2\Delta) \right\} \\ \frac{\partial c_A^*}{\partial \chi} &= \frac{\alpha^2 d}{4q\sqrt{D}} \left\{ d(1+4\alpha) - \sqrt{D} \right\} \\ \frac{\partial c_A^*}{\partial q} &= \frac{d}{2q\sqrt{D}} \left\{ \alpha\sqrt{D} + [4q - \alpha(1+4\alpha)d] \right\}\end{aligned}$$

where $\alpha = 2q/\chi$ and $D = d^2(1+4\alpha)^2 + 4d\Delta - 32dq + 4\Delta^2 (> 0)$. First, $\partial c_A^*/\partial d > 0$ because the second regularity condition (C.5) implies $d \leq \frac{2(8q-\Delta)}{(1+4\alpha)^2}$ and thus

$$16q - d(1+4\alpha)^2 - 2\Delta \geq 16q - \frac{2(8q-\Delta)}{(1+4\alpha)^2} \cdot (1+4\alpha)^2 - 2\Delta = 0.$$

Second, $\partial c_A^*/\partial \Delta < 0$ because

$$\begin{aligned}D - (d+2\Delta)^2 &= 8d \left\{ \alpha(1+2\alpha)d - 4q \right\} \geq 8d \left\{ \alpha(1+2\alpha) \cdot \frac{2(8q-\Delta)}{(1+4\alpha)^2} - 4q \right\} \\ &= -\frac{16d}{(1+4\alpha)^2} \left\{ \alpha(1+2\alpha)\Delta + 2(1+4\alpha+8\alpha^2)q \right\} < 0\end{aligned}$$

by the second regularity condition (C.5). Third, $\partial c_A^*/\partial \chi$ has the opposite sign of $d(\Delta - 8q) + \Delta^2$ because $D = \{d(1+4\alpha)\}^2 - 4\{d(\Delta - 8q) + \Delta^2\}$.

Lastly, we show $\partial c_A^*/\partial q > 0$. This is obvious when $4q \geq \alpha(1+4\alpha)d$. Suppose otherwise; i.e., $d > \frac{4q}{\alpha(1+4\alpha)}$. Then, a necessary and sufficient condition for $\partial c_A^*/\partial q > 0$ is

$$\alpha^2 D - [\alpha(1+4\alpha)d - 4q]^2 = 4(\alpha\Delta + 2q) \left\{ \alpha(d + \Delta) - 2q \right\}$$

is positive. This condition is true because

$$\alpha(d + \Delta) - 2q > \alpha \left\{ \frac{4q}{\alpha(1+4\alpha)} + \frac{4q}{2\alpha + 1} \right\} - 2q = \frac{2q}{(1+2\alpha)(1+4\alpha)} > 0$$

due to the supposition $d > \frac{4q}{\alpha(1+4\alpha)}$ and the first regularity condition (C.4).