

Generalized Coarse Matching

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Abstract

This paper analyzes the problem of matching two heterogeneous populations, such as men and women. If the payoff from a match exhibits complementarities, it is well known that, absent any friction, positive assortative matching is optimal. Coarse matching refers to a situation in which the populations are sorted into a finite number of classes and then randomly matched within these classes. We derive upper bounds on the fraction of the total efficiency loss of n -class coarse matching, which is proportional to $1/n^2$. Our result substantially enlarges the scope of matching problems in which the performance of coarse matching can be assessed.

JEL: D42, D82, D83, D86

1 Introduction

Consider a canonical matching problem: there are two heterogeneous populations of equal size. Agents within each population have the same preferences regarding the other population. For instance, one population may consist of men and a second of women. Within each population, agents differ by ability, beauty, education, etc., and each prefers agents from the other population who have higher ability, beauty, education, etc. If two agents (e.g., a man and woman) match, their payoffs depend on the characteristics of both partners.

It is well known that if a payoff function exhibits complementarities, it is optimal to match the two populations in a positive assortative fashion to maximize total match payoffs

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(e.g., Becker, 1973). That is, men with the best (worst) characteristics are matched with the women with the best (worst) characteristics. This method of matching is clearly better than randomly assigning men to women.

This paper focuses on an intermediate method of pairing two populations—namely, *coarse matching*. It proceeds by partitioning the population into n classes and then randomly matching individuals within each class. We ask: how much of total surplus is captured by coarse matching (henceforth, CM) when compared with the total surplus generated by positive assortative matching (henceforth, PAM)?¹ Before presenting the result, let us explain the importance of the question.

Note that PAM requires a matchmaker (planner) who knows the actual type of each member of each population. When the populations are large, this is a strong informational requirement. In practice, there is a cost of acquiring information on agents' types, but this cost is not modeled in typical analyses. We would think that the cost of acquiring perfect information is significantly greater than the cost of acquiring coarse information. Given these costs, should the social planner implement PAM or settle for CM? The answer depends on whether the efficiency gain of using PAM over CM is small or large. If it is small, it may not justify the cost of acquiring this additional information.²

This question was first addressed by McAfee (2002). He considers coarse matching, with each population partitioned into *exactly* two classes. He shows that when the match payoff function takes a multiplicative form and distributions of attributes satisfy certain hazard rate conditions, CM can lose no more than half of the total surplus generated by PAM.

We tackle this problem and investigate the performance of coarse matching (i.e., the fraction of the efficiency loss). The main result shows that for each n , we can construct an n -class CM with an upper bound on its efficiency loss. In particular, this upper bound is an expression that is proportional to $1/n^2$. We also show by example that this upper bound is tight. The coefficient of the upper bound has two parts: the first part measures the efficiency loss due to the variation in the complementarity (cross derivative) of the match payoff; and the second part captures the efficiency loss due to agents' type distributions. Our result also

¹In this paper, by PAM, we mean perfect sorting, although CM also involves positive assortative matching in a very coarse sense.

²This is not the only justification for CM. For example, McAfee (2002) argues, “The use of a continuum of priorities is not feasible in many circumstances—using many priorities makes the scheme unwieldy to administer and opaque to consumers. Moreover, if the priority prices are determined by bidding, as is natural, the auction process will be complex and expensive to operate when there are many service classes.” Hoppe, Moldovanu, and Ozdenoren (2010) suggest that “[t]hese costs may take the form of: communication, complexity (or menu), and evaluation costs for the intermediary (who needs more detailed information about the environment in order to implement a fine scheme), and for the agents (who need precise information about their own and others' attributes in order to optimally respond to a fine scheme), or higher production costs for firms offering different qualities.”

separates these two effects.

Our main result generalizes McAfee’s result along several dimensions by allowing: 1) arbitrary match payoff functions that are complementary in agents’ characteristics; 2) almost any distributions of types; and 3) an arbitrary fixed number of classes. Note that most results in the literature are derived by assuming that match payoff is multiplicative in agents’ types. Our paper, hence, substantially enlarges the class of matching problems to include almost any in which the performance of CM can be assessed.

While Chebyshev’s sum inequality plays the central role in McAfee (2002), one of our technical contributions is the novel use of the so-called “Grüss’s inequality” to establish our result, which could be useful for future studies.

RELATED LITERATURE. Wilson (1989) considers a mathematically equivalent model and shows that the efficiency loss of using n -class CM converges to zero at a rate of $O(1/n^2)$. Nevertheless, the convergence rate is not quite informative for understanding the performance of pricing schedules with a given number of priority groups, especially when this number is not large. McAfee (2002) rephrases that model as a two-sided coarse matching model. He proves that under a restriction to a certain class of distributions, 2-class CM has no more than half of the efficiency loss. Hoppe, Moldovanu, and Ozdenoren (2010) further refine McAfee’s (2002) result with a tighter bound and derive new bounds for other classes of distributions. By applying these results to a monopolistic pricing problem with private information, they analyze the performance of coarse matching in terms of output, revenue and agents’ welfare. However, they still restrict their analysis to 2-class CM with multiplicative payoff functions.

Our paper proceeds as follows: In Section 2, we introduce the formal model, notations and assumptions. In Section 3, we present our main finding. Section 4 concludes. The Appendix contains the details of proofs.

2 The Model

A two-sided market consists of two equal-sized populations of agents. For clarity, we refer to the agents of each population as men and women, each of whom is characterized by type. Denote the types as x and y , respectively. Agents’ types are distributed over $[0, 1]$ according to distribution functions $F(x)$ and $G(y)$.³ Throughout the paper, we assume that corresponding density functions $f(x)$ and $g(y)$ are continuous and strictly positive over $(0, 1)$. We denote the mean μ_x and μ_y , respectively.

Each agent is assumed to be matched with one agent from the other side of the market—

³Our results hold for any positive type space that is an interval with finite length. Without loss, we just assume $[0, 1]$ for simplicity.

that is, one man may marry only one woman. For matched agents with types x and y , the match payoff is $m(x, y)$, which is assumed to be strictly complementary in x and y and smooth enough—i.e., $m_{12} = \frac{\partial^2}{\partial x \partial y} m(x, y) > 0$. We assume that m_{12} is bounded by \underline{m} , \bar{m} from below and above.⁴

We are interested in the overall match payoff of all agents. It is well known in the literature (e.g., Becker, 1973) that if a match payoff function exhibits complementarity in agents' types, the optimal allocation involves PAM. In other words, the highest-type man matches the highest-type woman, and the second-highest-type man matches the second-highest-type woman, and so on. Denote the agent's percentile as i indexed by the rank of his/her type in each population. With a slight abuse of notation, let $x(i)$ and $y(i)$ be agents' types given their percentiles, respectively, with $x(i) = F^{-1}(i)$ and $y(i) = G^{-1}(i)$. Apparently, i is uniformly distributed over $[0, 1]$.

The total payoff from PAM is

$$U_\infty = \int_0^1 m(x(i), y(i)) di.$$

By contrast, the total payoff from random matching is

$$U_1 = \int_0^1 \int_0^1 m(x(i), y(j)) didj.$$

Clearly, $U_\infty > U_1$.

To interpret what follows, it is instructive to think of a matchmaker who can manipulate a certain type of man to match a certain type of woman. The goal of the matchmaker is to maximize the total payoff of all agents. However, to achieve PAM, the matchmaker has to know an inordinate amount of information—i.e., the true type of each agent. Alternatively, the matchmaker can divide the population into n classes—i.e., create n submarkets for men and women to meet. Within each submarket, agents match randomly. This matching scheme, denoted as *n-class coarse matching*, requires much less information than PAM since the matchmaker need only know the interval to which an agent belongs instead of his/her exact type.

Now, consider n -class CM, which partitions agents into n classes (submarkets)—i.e., $I_k = [i_{k-1}, i_k]$ with $i_0 = 0$ and $i_n = 1, k = 1, \dots, n$. For each submarket I_k , the match payoff

⁴The strict complementarity is assumed for technical convenience to estimate the match surplus of PAM over random matching, $U_\infty - U_1$ defined later in the paper, which is then used to measure efficiency loss of CM in the main theorem. Note that $U_\infty - U_1$ is independent of the formation of any CM. It can be directly calculated as a parameter of the matching problem without this assumption.

is

$$\frac{1}{(i_k - i_{k-1})^2} \int_{I_k} \int_{I_k} m(x(i), y(j)) didj.$$

It is then straightforward to see that a set of cutoff points $\{i_k\}_{k=1}^{n-1}$ uniquely determines an n -class CM scheme. The total payoff U_n is

$$U_n = \sum_{k=1}^n \frac{1}{(i_k - i_{k-1})^2} \int_{I_k} \int_{I_k} m(x(i), y(j)) didj.$$

Apparently, U_n is a function of $\{i_k\}_{k=1}^{n-1}$. Hence, whenever we invoke U_n , it implicitly means that there exists an n -class CM scheme with cutoff points $\{i_k\}_{k=1}^{n-1}$ and total match payoff U_n . If $n = 1$, CM is random matching.

To explore the performance of n -class CM relative to PAM, we measure the percentage efficiency loss of n -class CM relative to PAM by

$$L_n = \frac{U_\infty - U_n}{U_\infty - U_1}.$$

The denominator is the total surplus of PAM over random matching. The numerator is the efficiency loss of n -class CM relative to PAM.

3 Main Result

Before we present our main result, it is instructive to discuss McAfee's (2002) result.

Theorem 3.1. (McAfee, 2002) If $m(x, y) = x \cdot y$ and distribution functions F and G satisfy

1. $F(x)/f(x)$ and $G(y)/g(y)$ are increasing, and
2. $[1 - F(x)]/f(x)$ and $[1 - G(y)]/g(y)$ are decreasing,

with cutoff point i_1 such that either $x(i_1) = \mu_x$ or $y(i_1) = \mu_y$, then,

$$L_2 \leq \frac{1}{2}.$$

This result shows that for 2-class CM, the efficiency loss is no more than one half of the total efficiency, as long as F and G satisfy the two hazard-rate conditions in Theorem 3.1. Note that CM, decided in this way, is not necessarily optimal. The main point is that there exists at least some 2-class CM that doesn't lose much total efficiency. In other words, 2-class CM can perform decently relative to PAM.

However, Theorem 3.1 holds only when match payoff functions are multiplicative separable—i.e., $m(x, y) = x \cdot y$, distributions satisfy two hazard-rate conditions, and the number of classes is two. In fact, most papers (e.g., McAfee (2002) and Hoppe et al. (2010)) in this literature focus mainly on the multiplicative separable match payoff functions. In contrast, our main result below holds without any of these restrictions and, thus, it can be applied to almost any matching problem. In order to establish it, we first derive Lemma 1 (the proof is left to the Appendix), which links matching problems with arbitrary match payoff functions to ones with the multiplicative separable match payoff function. For convenience, we denote U_∞ and U_n as u_∞ and u_n correspondingly when $m(x, y) = xy$.

Lemma 1.

$$L_n \leq \frac{\bar{m} u_\infty - u_n}{\underline{m} u_\infty - u_1}.$$

Coefficient \bar{m}/\underline{m} in Lemma 1 directly captures the effect on the matching surplus due to the change in complementarity (m_{12}) of the match payoff. Lemma 1 separates this effect from the formation of coarse matching—i.e., the number of classes and the choice of cutoff points. Note that when $m(x, y) = xy$, complementarity is constant; hence, $\bar{m}/\underline{m} = 1$. One immediate result from Theorem 3.1 and Lemma 1 is:

Corollary 3.2. If F and G satisfy the same conditions with the same cutoff point as in Theorem 3.1, we have

$$L_2 \leq \frac{1}{2} \frac{\bar{m}}{\underline{m}}.$$

We now present our *main result*, which generalizes Theorem 3.1.

Denote a, b and A, B as lower and upper bounds of densities f, g , respectively—i.e., $a \leq f \leq A$ and $b \leq g \leq B$ where $0 \leq a, b, A, B < \infty$.

Theorem 3.3. There is an n -class CM scheme satisfying

$$L_n \leq \frac{\bar{m}}{\underline{m}} \frac{\beta}{n^2}$$

where $\beta = \min \left\{ 3AB, \frac{AB}{ab} \right\}$ with the cutoffs points $\{i_k\}_{k=1}^{n-1}$ associated with coefficient

1. $3AB$, satisfying $(x(i_k) - x(i_{k-1}))(y(i_k) - y(i_{k-1})) = 1/n^2$ with $k = 1$ to whenever possible up to $k \leq n - 1$.
2. $(AB)/(ab)$, satisfying $i_k = k/n$, where $k = 1, \dots, n - 1$.

Example 3.1. Suppose that $m(x, y) = e^{\frac{1}{4}xy}$ and x and y are uniformly distributed over $[0, 1]$. It is easy to check that $m_{12} = \frac{1}{16}e^{\frac{1}{4}xy}(xy + 4) \in \left[\frac{1}{4}, \frac{5}{16}e^{\frac{1}{4}}\right]$ and $\overline{m}AB/\underline{m}ab \approx 1.61$. By Theorem 3.3,

$$L_n \leq \frac{1.61}{n^2}.$$

SKETCH OF THE PROOF. We sketch the proof of Theorem 3.3 here (see the Appendix for the complete proof). Lemma 1 allows us to estimate the upper bound by focusing on $(u_\infty - u_n)/(u_\infty - u_1)$, in which $m(x, y) = xy$. Since $u_\infty - u_n$ can be represented as a weighted sum of surpluses of all submarkets, our goal is to derive the upper bound of the surplus of PAM over random matching of each submarket and aggregate them together. One key tool used to estimate such surpluses is Grüss’s inequality:

Lemma 2 (Grüss’s Inequality).

$$\left| \int_{I_k} \frac{x(i)y(i)}{i_k - i_{k-1}} di - \int_{I_k} \int_{I_k} \frac{x(i)y(j)}{(i_k - i_{k-1})^2} didj \right| \leq \frac{1}{4} \left(\max_{I_k} x - \min_{I_k} x \right) \left(\max_{I_k} y - \min_{I_k} y \right).$$

Grüss’s inequality directly gives the upper bound of the total surplus of PAM over random matching when $m(x, y) = xy$. By Grüss’s inequality, we prove that the surplus of each submarket is bounded from above by a constant proportional to $1/n^2$. By estimating $u_\infty - u_1$ with bounds of densities, we establish the upper bound in Theorem 3.3.

In Theorem 3.3, depending on the properties of distributions, the upper bound is tighter with a certain coefficient. We provide two coefficients— $\{3AB, (AB)/(ab)\}$ —to choose the tighter one. For each coefficient, there exists a corresponding set of cutoff points to ensure the inequality in Theorem 3.3.

Similar to Theorem 3.1, the set of the cutoff points associated with a coefficient is not necessary to be the optimal n -class CM scheme, which requires optimization over i_k . Loosely speaking, the n -class CM scheme in Theorem 3.3 can be considered the “worst case” since it shows that n -class CM with cutoff points chosen in a mechanical way can have an efficiency loss that is no more than a certain threshold. Although the coefficients in Theorem 3.3 involve the bounds of densities f and g , the boundness conditions of f and g are very mild.⁵

Theorem 3.3 answers how the upper bound of efficiency loss changes over the number of classes. It separates the effects due to the properties of the market—i.e., $m(x, y)$, F , G and the fineness of the partition of the populations. Moreover, Theorem 3.3 holds for almost any payoff functions and distributions.

⁵If the upper bounds of f and g are infinite, we can define A and B as essential suprema, instead. Then, A and B must be finite due to the existence of means. The proof of Theorem 3.3 still holds.

It is not surprising that Theorem 3.3 implies that the upper bound is $O(1/n^2)$. Wilson (1989) shows that the efficiency loss converges to zero at the rate $1/n^2$. The intuition of the rate $1/n^2$ is: the efficiency loss due to mismatch grows large with the size of the submarket. To limit the efficiency loss, the population on each side should be divided into classes with similar sizes, roughly around $1/n$. Then, the matching of all the classes yields the efficiency loss proportional to $1/n^2$. However, the convergence rate result alone is not very helpful for understanding the performance of CM relative to PAM, especially when the number of classes is not very large, which is often the case in practice. Thus, our upper bound result is more informative and useful in evaluating CM.

As a comparison, Theorem 3.1 provides an elegant upper bound when restricting to a particular class of distributions satisfying hazard-rate conditions. A non-trivial excise can extend the upper bound in Theorem 3.1 to n -class, which is $O(1/n)$. Similarly, by restricting to a class of distributions with certain properties, one can derive tighter upper bounds by using the argument in Theorem 3.3 with particularly refined Grüss-type inequalities. In the next example, we illustrate that the upper bound in Theorem 3.3 can be quite tight.

Example 3.2. Consider $F(x) = G(x) = x$. Then, $f = g = 1$. By Theorem 3.3, we have $L_n \leq 1/n^2$. It is easy to verify that the optimal n -class CM scheme has $i_k = k/n$ with total match payoff U_n^* such that

$$\frac{U_\infty - U_n^*}{U_\infty - U_1} = \frac{1}{n^2}.$$

This example shows that the upper bound derived in Theorem 3.3 is tight, in the sense that, for some distributions, the upper bound of efficiency loss is the minimum that can be achieved by the optimal n -class CM scheme.

4 Concluding Remarks

We analyze the performance of n -class CM with heterogeneous agents relative to the optimal matching scheme that requires PAM. We provide upper bounds on the efficiency loss for arbitrary n -class CM. In contrast to most existing results, our main result can be applied to almost any matching problem. Moreover, such a generalization is also crucial when applying the matching results to the models with private information—e.g., monopolistic pricing (Maskin and Riley, 1984; Mussa and Rosen, 1978) and regulation problems (Laffont and Tirole, 1986), as in Hoppe et al. (2010) and McAfee (2002) (see, also, Rogerson (2003) and Chu and Sappington (2007)). In these analyses, the matching problems are generated endogenously by the optimal contracts. Thus, the generated match payoff functions and

distributions may often violate the specific conditions assumed in previous results. Our result circumvents this weakness.

Appendix

Proof of Lemma 1. For any arbitrary $I_k = [i_{k-1}, i_k]$,

$$\begin{aligned}
& \int_{I_k} m(x(i), y(i)) di - \frac{1}{i_k - i_{k-1}} \int_{I_k} \int_{I_k} m(x(i), y(j)) didj \\
&= \frac{1}{i_k - i_{k-1}} \int_{I_k} \int_{I_k} \int_j^i \int_{i_{k-1}}^i m_{12}(x(p), y(q)) x'(p) y'(q) dpdqdidj \\
&\geq \frac{\underline{m}}{i_k - i_{k-1}} \int_{I_k} \int_{I_k} (y(i) - y(j)) (x(i) - x(i_{k-1})) didj \\
&= \underline{m} \left(\int_{I_k} x(i) y(i) di - \frac{1}{i_k - i_{k-1}} \int_{I_k} \int_{I_k} x(i) y(j) didj \right).
\end{aligned}$$

Similarly, we also have

$$\begin{aligned}
& \int_{I_k} m(x(i), y(i)) di - \frac{1}{i_k - i_{k-1}} \int_{I_k} \int_{I_k} m(x(i), y(j)) didj \\
&\leq \bar{m} \left(\int_{I_k} x(i) y(i) di - \frac{1}{i_k - i_{k-1}} \int_{I_k} \int_{I_k} x(i) y(j) didj \right).
\end{aligned}$$

Note that

$$U_\infty - U_n = \sum_{k=1}^n (i_k - i_{k-1}) \left(\int_{I_k} \frac{m(x(i), y(i))}{i_k - i_{k-1}} di - \frac{1}{(i_k - i_{k-1})^2} \int_{I_k} \int_{I_k} m(x(i), y(j)) didj \right).$$

Hence,

$$\begin{aligned}
U_\infty - U_n &\leq \bar{m}(u_\infty - u_n), \text{ and} \\
U_\infty - U_1 &\geq \underline{m}(u_\infty - u_1).
\end{aligned}$$

That is,

$$\frac{U_\infty - U_n}{U_\infty - U_1} \leq \frac{\bar{m} u_\infty - u_n}{\underline{m} u_\infty - u_1}. \quad \square$$

We use the following result from Mitrovic et al. (1993) in the proof of our main result.

Lemma 3.

$$\frac{1}{i_k - i_{k-1}} \int_{I_k} x(i) y(i) di - \frac{1}{(i_k - i_{k-1})^2} \int_{I_k} \int_{I_k} x(i) y(j) didj = \frac{1}{12} (i_k - i_{k-1})^2 x'(\varepsilon) y'(\eta),$$

where $\varepsilon, \eta \in I_k$.

Proof of Theorem 3.3. By Lemma 1, we focus on $(u_\infty - u_n) / (u_\infty - u_1)$. Then,

$$u_\infty - u_n = \sum_{k=1}^n (i_k - i_{k-1}) \left[\int_{I_k} \frac{x(i) y(i)}{i_k - i_{k-1}} di - \int_{I_k} \int_{I_k} \frac{x(i) y(j)}{(i_k - i_{k-1})^2} didj \right].$$

We derive the two coefficients in two parts.

Part 1: We apply Lemma 2 to $u_\infty - u_n$,

$$\begin{aligned} u_\infty - u_n &\leq \frac{1}{4} \sum_{k=1}^n (i_k - i_{k-1}) (x(i_k) - x(i_{k-1})) (y(i_k) - y(i_{k-1})) \\ &\leq \frac{1}{4} \frac{1}{n^2}. \end{aligned} \tag{1}$$

To show the second inequality above, we choose i_k such that

$$(x(i_k) - x(i_{k-1})) (y(i_k) - y(i_{k-1})) = \frac{1}{n^2},$$

starting from $k = 1$ up to some $\tilde{k} - 1$ such that $(1 - x(i_{\tilde{k}})) (1 - y(i_{\tilde{k}})) \leq 1/n^2$. Let $X_k = x(i_k) - x(i_{k-1})$ and $Y_k = y(i_k) - y(i_{k-1})$. Without loss, we assume that for all $k \leq n - 1$, $X_k Y_k = \frac{1}{n^2}$. Then, $X_n Y_n = (1 - \sum_{k=1}^{n-1} X_k) (1 - \sum_{k=1}^{n-1} Y_k)$. It is not difficult to show that the maximum of $(1 - \sum_{k=1}^{n-1} X_k) (1 - \sum_{k=1}^{n-1} Y_k)$ is less than $1/n^2$ for any non-negative X_k and Y_k satisfying $X_k Y_k = 1/n^2$.

Part 2: By Lemma 3,

$$u_\infty - u_n = \frac{1}{12} \sum_{k=1}^n (i_k - i_{k-1})^3 x'(\varepsilon_k) y'(\eta_k),$$

where $\varepsilon_k, \eta_k \in I_k$. Note that $x'(\varepsilon_k) = F^{-1'}(\varepsilon_k) \leq 1/a$. Similarly, $y'(\eta_k) \leq 1/b$. Let $i_k - i_{k-1} = 1/n$,

$$u_\infty - u_n \leq \frac{1}{12abn^2}. \tag{2}$$

By Lemma 3,

$$u_\infty - u_1 = \frac{1}{12} x'(\varepsilon) y'(\eta) \geq \frac{1}{12AB}. \tag{3}$$

By (1), (2) and (3), we establish the upper bound in Theorem 3.3. □

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