

Exponentially fast dynamics of chaotic many-body systemsFausto Borgonovi,^{1,2} Felix M. Izrailev,^{3,4} and Lea F. Santos⁵¹*Dipartimento di Matematica e Fisica and Interdisciplinary Laboratories for Advanced Materials Physics, Università Cattolica, via Musei 41, 25121 Brescia, Italy*²*Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, via Bassi 6, I-27100, Pavia, Italy*³*Instituto de Física, Benemérita Universidad Autónoma de Puebla, Apartado Postal J-48, Puebla 72570, Mexico*⁴*Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824-1321, USA*⁵*Department of Physics, Yeshiva University, New York, New York 10016, USA*

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We demonstrate analytically and numerically that in isolated quantum systems of many interacting particles, the number of many-body states participating in the evolution after a quench increases exponentially in time, provided the eigenstates are delocalized in the energy shell. The rate of the exponential growth is defined by the width Γ of the local density of states and is associated with the Kolmogorov-Sinai entropy for systems with a well-defined classical limit. In a finite system, the exponential growth eventually saturates due to the finite volume of the energy shell. We estimate the timescale for the saturation and show that it is much larger than \hbar/Γ . Numerical data obtained for a two-body random interaction model of bosons and for a dynamical model of interacting spin-1/2 particles show excellent agreement with the analytical predictions.

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I. Introduction. After decades of intensive studies, the term “quantum chaos” [1–10] has become widely disseminated and accepted in modern physics. Originally it referred to quantum systems whose classical counterparts are chaotic. Paradigmatic examples are the kicked rotor model (KRM) [1,2] and billiard models [3–5], both of which reveal quantum signatures of classical chaos [11,12]. It was conjectured and numerically proved [4,5] that quantum chaos might be quantified by specific properties of the fluctuations of energy spectra. In particular, it was found that in chaotic systems, the distribution of spacings between neighboring energy levels follows closely the Wigner surmise [13], in contrast with the Poisson dependence that emerges in integrable systems.

Throughout the development of one-body quantum chaos, dynamics has played a crucial role. Numerical KRM studies [1,2] discovered the unexpected existence of two timescales associated with the quantum-classical correspondence. It was confirmed that a complete correspondence between the quantum and classical behavior occurs only on a tiny timescale according to the Ehrenfest theorem. It was analytically shown in Ref. [14] that this timescale is given by $t_E \simeq \lambda^{-1} \ln(I/\hbar)$, where I represents a characteristic action and λ is the classical Lyapunov exponent. However, numerical data reported and discussed in Refs. [1,2] revealed the existence of a much larger timescale on which the behavior of classical and quantum global observables are equivalent. This timescale was found to be $t_D \propto D/\hbar^2$, where D is the classical diffusion coefficient in the momentum space. After such time and in contrast with the classical case, quantum diffusion ceases. This phenomenon, called dynamical localization, was explained by the localization of the eigenstates in momentum space according to the relation $\ell \propto D$, where ℓ is the localization length [2,15]. It was later argued that the dynamical localization found in the

KRM can be also thought of in terms of Anderson localization in pseudorandom potentials [16].

Contrary to one-body quantum chaos, in quantum many-body systems (MBSs), level statistics is less informative than the structure of the eigenstates in a physically chosen basis [10,17]. It is now understood, for example, that the relaxation of a quantum MBS to its thermal state requires the presence of chaotic eigenstates [8–10,18]. The relaxation of a quantum MBS in the thermodynamic limit has been discussed [19], but the timescale on which it occurs in finite systems is still an open question. To address this problem, we analyze the relaxation of observables of quantum MBSs in the many-body space.

We consider the quench dynamics described by a Hamiltonian $H = H_0 + V$ in the region of parameters where the eigenstates are fully delocalized in the energy shell defined by the interparticle interaction V [17,20–23]. Specifically, we prepare the system in a single (unperturbed) eigenstate of H_0 and study how the state spreads in the unperturbed many-body basis due to V . With the use of a semi-analytical approach, we show that the effective number of unperturbed states participating in the dynamics of quantum MBSs increases exponentially in time.

We find that the exponential growth saturates at a time much larger than the characteristic time \hbar/Γ of the initial state decay, where Γ is the width of the local density of states (LDOS). [The LDOS describes the energy distribution of the initial state. It is obtained by projecting the initial state on the energy eigenbasis.] We discuss the physical meaning of this novel timescale in connection with the quantum-classical correspondence for chaotic MBSs and with the problem of thermalization in isolated quantum MBSs. Our analytical estimates are fully confirmed by numerical data obtained via exact diagonalization for two different systems: a model

of randomly interacting bosons and a one-dimensional (1D) system of spins 1/2 with deterministic couplings.

II. Models. In both models, H_0 describes the noninteracting particles (or quasiparticles), while their interaction is contained in V . The first model represents N identical bosons occupying M single-particle levels specified by random energies ϵ_s with a mean spacing $\langle \epsilon_s - \epsilon_{s-1} \rangle = 1$ setting the energy scale. The Hamiltonian reads

$$H = \sum_s \epsilon_s a_s^\dagger a_s + \sum_{s_1 s_2 s_3 s_4} V_{s_1 s_2 s_3 s_4} a_{s_1}^\dagger a_{s_2}^\dagger a_{s_3} a_{s_4}, \quad (1)$$

where a_s^\dagger (a_s) is the creation (annihilation) operator on level s , and the two-body matrix elements $V_{s_1 s_2 s_3 s_4}$ are random Gaussian entries with zero mean and variance v^2 . The interaction conserves the number of bosons and connects many-body states that differ by changing at most two particles. This two-body interaction (TBRI) random model was introduced in Refs. [24,25] to model nuclear systems. It has been extensively studied for fermions [20,26] and bosons [27]. It has also been used to describe nonrandom systems, such as the Lieb-Liniger model [28] largely investigated experimentally [29]. The unperturbed many-body eigenstates $|k\rangle$ of $H_0 = \sum_k \mathcal{E}_k |k\rangle \langle k|$ are obtained by all possible combinations of N bosons in M single-particle energy levels according to standard statistical rules. This generates $\mathcal{D} = \frac{(N+M-1)!}{N!(M-1)!}$ unperturbed many-body states. The eigenstates $|\alpha\rangle$ of the Hamiltonian $H = \sum_\alpha E^\alpha |\alpha\rangle \langle \alpha|$ are represented in terms of the states $|k\rangle$ as $|\alpha\rangle = \sum_k C_k^\alpha |k\rangle$.

The other model studied has no random terms. It describes a dynamical system of interacting spins-1/2 on a 1D lattice of length L . Spin systems are intensively studied in experiments with nuclear magnetic resonance platforms [30] and ion traps [31], as well as similar systems with cold atoms [32]. The Hamiltonians H_0 and V are given by

$$H_0 = \frac{J}{4} \sum_s (\sigma_s^x \sigma_{s+1}^x + \sigma_s^y \sigma_{s+1}^y + \Delta \sigma_s^z \sigma_{s+1}^z), \quad (2)$$

$$V = \lambda \frac{J}{4} \sum_s (\sigma_s^x \sigma_{s+2}^x + \sigma_s^y \sigma_{s+2}^y + \Delta \sigma_s^z \sigma_{s+2}^z), \quad (3)$$

where $\sigma_s^{x,y,z}$ are the Pauli matrices on site s . The coupling constant $J = 1$ sets the energy scale, Δ is the anisotropy parameter, and λ is the ratio between nearest-neighbor and next-nearest-neighbor couplings [33]. The Hamiltonian conserves the total spin in the z direction, $\mathcal{S}^z = \sum_{s=1}^L \sigma_s^z / 2$, which is here fixed to $\mathcal{S}^z = -1$, where L is even and the number of upspins (excitations) is given by $N = L/2 - 1$. The dimension of the Hamiltonian matrix is $\frac{L!}{N!(L-N)!}$. When $V = 0$, the model is integrable, while as λ increases, it becomes chaotic [17].

Basic relations. We analyze the wave packet dynamics in the unperturbed basis $|k\rangle$ after switching on the interaction V . The system is initially prepared in a particular unperturbed state $|k_0\rangle$,

$$|\psi(0)\rangle = \sum_\alpha C_{k_0}^\alpha |\alpha\rangle. \quad (4)$$

The probability to find the evolved state in any basis state $|k\rangle$ at the time t is

$$P_k(t) = |\langle k | \psi(t) \rangle|^2 = \sum_{\alpha, \beta} C_{k_0}^{\alpha*} C_k^\alpha C_{k_0}^\beta C_k^{\beta*} e^{-i(E^\beta - E^\alpha)t}, \quad (5)$$

which can be written as the sum of a diagonal part, $P_k^d = \sum_\alpha |C_{k_0}^\alpha|^2 |C_k^\alpha|^2$, and an oscillating time-dependent part, $P_k^f(t) = \sum_{\alpha \neq \beta} C_{k_0}^{\alpha*} C_k^\alpha C_{k_0}^\beta C_k^{\beta*} e^{-i(E^\beta - E^\alpha)t}$. After a long time and assuming a nondegenerate spectrum, P_k^f cancels out on average and only the diagonal part P_k^d survives.

With $P_k(t)$, we construct the quantity of our main interest, the number of principal components,

$$N_{pc}(t) = \left\{ \sum_k [P_k^d + P_k^f(t)]^2 \right\}^{-1}, \quad (6)$$

also known as participation ratio [34]. It measures the effective number of unperturbed states $|k\rangle$ that compose the evolved wave packet. For weak interaction, $N_{pc}(t)$ oscillates in time. Our focus is, however, on strong values of V , where $N_{pc}(t)$ increases smoothly and eventually saturates to its infinite time average given by

$$\overline{N_{pc}^\infty} = \left[2 \sum_k (P_k^d)^2 - \sum_\alpha |C_{k_0}^\alpha|^4 \sum_k |C_k^\alpha|^4 \right]^{-1}. \quad (7)$$

This determines the total number of unperturbed many-body states inside the energy shell.

III. Dynamics in many-body space. A distinctive property of the dynamics of a quantum MBS is that it cannot be described as either ballistic or diffusive in the many-body space. A pictorial demonstration of how the initial state spreads in the many-body space is given in the Supplemental Material [35]. Specifically, on a small timescale, only the basis states directly coupled to the initial state are excited. Their number is much smaller than the total number of basis states, due to the sparse structure of the Hamiltonian matrix. As time passes more basis states are populated inside the shell, until its ergodic filling. This takes place provided the perturbation V is sufficiently strong so that the eigenstates of H are delocalized in the energy shell.

To describe the time dependence of $N_{pc}(t)$, we develop a cascade model to monitor the flow of probability to find the system in specific unperturbed states at different time steps. This is done by dividing the dynamical process in different time intervals associated with different sets of basis states (classes). At $t = 0$, only the \mathcal{M}_0 class is not empty: it has one element, which is the initial state $|k_0\rangle$. In the next time step, all states having a nonzero coupling with the initial basis state are populated; that is, the first class \mathcal{M}_1 contains the basis states $|k\rangle$ for which $\langle k_0 | V | k \rangle \neq 0$. The second class \mathcal{M}_2 consists of those states which have nonzero matrix elements with all states from the first class. In the same manner, one can define all classes in the many-body space.

For an infinite number of particles, there is an infinite hierarchy of equations describing the flow of probability from one class to the next one. However, for the values of N and M accessible to our computers, the number of states in the second class practically coincides with \mathcal{D} , so only two classes

can be considered [35]. As shown below, this is indeed a good approximation.

Let us define the probability to find the system in class \mathcal{M}_0 , as $W_0(t) \equiv P_{k_0}(t)$. This is the survival probability of the initial state. The probability for being in the class \mathcal{M}_1 is $W_1(t) \equiv \sum_{k \in \mathcal{M}_1} P_k(t)$. Neglecting the back flow to the initial state, we can write the following set of rate equations [35]:

$$\begin{aligned} \frac{dW_0}{dt} &= -\Gamma(W_0 - \overline{W_0^\infty}), \\ \frac{dW_1}{dt} &= -\Gamma(W_1 - \overline{W_1^\infty}) + \Gamma(W_0 - \overline{W_0^\infty}), \end{aligned} \quad (8)$$

where the infinite time averages are $\overline{W_0^\infty} = \sum_\alpha |C_{k_0}^\alpha|^4$ and $\overline{W_1^\infty} = \sum_{k \in \mathcal{M}_1} \sum_\alpha |C_{k_0}^\alpha|^2 |C_k^\alpha|^2$.

The decay rate Γ corresponds to the width of the LDOS,

$$F_{k_0}(E) = \sum_\alpha |C_{k_0}^\alpha|^2 \delta(E - E^\alpha), \quad (9)$$

which is obtained by projecting the initial state $|k_0\rangle$ onto the energy eigenbasis. It was introduced in nuclear physics to describe the relaxation of excited heavy nuclei [36], where it is known as ‘‘strength function.’’

The solution of Eq. (8) gives

$$\begin{aligned} W_0(t) &= e^{-\Gamma t} (1 - \overline{W_0^\infty}) + \overline{W_0^\infty}, \\ W_1(t) &= \Gamma t e^{-\Gamma t} (1 - \overline{W_0^\infty}) + \overline{W_1^\infty} (1 - e^{-\Gamma t}). \end{aligned} \quad (10)$$

With the expressions (10) one can derive the time dependence for $N_{pc}(t)$,

$$N_{pc}(t) \simeq \left[\sum_n W_n^2 / \mathcal{N}_n \right]^{-1} \simeq [W_0^2 + W_1^2 / \mathcal{N}_1]^{-1} \sim e^{2\Gamma t}, \quad (11)$$

where \mathcal{N}_n is the number of states contained in the n th class. This result shows that the number of basis states effectively participating in the evolution of the wave packet increases exponentially in time with the rate 2Γ . For a finite number of particles, this growth lasts until the saturation given by Eq. (7). We note that exponential instability was also studied in Ref. [37], where the number of harmonics of the Wigner function was shown to increase exponentially fast in time.

A. Results for the TBRI model. To verify the validity of our approach, we compare in Figs. 1(a) and 1(b) the numerical data for $W_0(t)$ and $W_1(t)$ with Eqs. (10). The chosen v is such that the eigenstates are strongly chaotic and extended in the energy shell [23]. The value of Γ used in the analytical expressions is obtained by fitting the numerical curve for $W_0(t)$. The agreement between numerical and analytical results is very good for the entire duration of the evolution, up to the

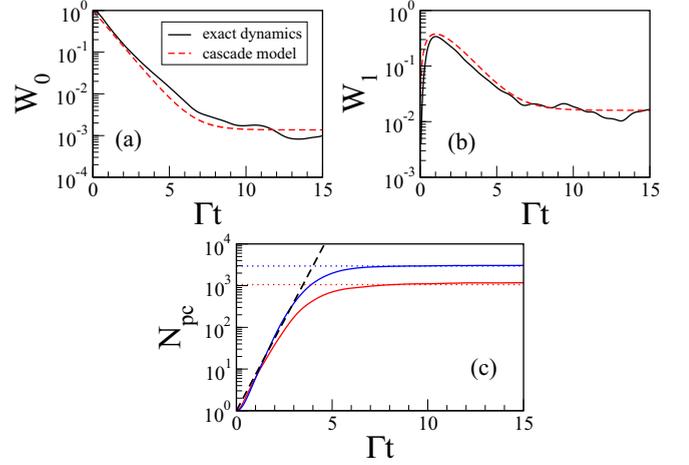


FIG. 1. TBRI model: Numerical data for $W_0(t)$ (a) and $W_1(t)$ (b) are shown by solid curves and compared with the analytical expressions (10) (dashed curves). The parameters are $N = 6$, $M = 11$, and $v = 0.4$ (chaotic regime). In the initial state $|\psi(0)\rangle = (a_5^\dagger)^6|0\rangle$ all particles initially occupy the fifth single-particle level. The exponential rate $\Gamma = 2.8$ is obtained by fitting $W_0(t)$. (c) Growth in time of N_{pc} for two initial conditions; from top to bottom: $|\psi(0)\rangle = (a_4^\dagger)^6|0\rangle$ and $|\psi(0)\rangle = (a_5^\dagger)^6|0\rangle$. The dashed line is $e^{2\Gamma t}$. Horizontal dotted lines are the analytical estimates given by Eq. (7). Average over 50 random realizations.

saturation given by $\overline{W_0^\infty}$ and $\overline{W_1^\infty}$. These results confirm that the back flow can indeed be neglected and that one can take into account two classes only.

In Fig. 1(c) we show the evolution of the number of principal components N_{pc} . The numerical data (solid curve) corroborate the analytical prediction (dashed curve) from Eq. (11), namely, the exponential behavior, $N_{pc}(t) \sim e^{2\Gamma t}$.

Our data manifest the existence of two timescales. The first one, $t_\Gamma \simeq 1/\Gamma$, corresponds to the characteristic decay time of $W_0(t)$, as shown in Eq. (10). The second, t_S , is the timescale for the saturation of the dynamics and can be estimated from $e^{2\Gamma t} \simeq \overline{N_{pc}^\infty}$, which gives

$$t_S \simeq \ln(\overline{N_{pc}^\infty})/2\Gamma. \quad (12)$$

Assuming a Gaussian shape for both the density of states and the LDOS [35], we show that the maximal value of $\overline{N_{pc}^\infty}$ is

$$N_{pc}^{\max} = \eta \sqrt{1 - \eta^2} \mathcal{D}, \quad (13)$$

where $\eta = \Gamma/\sigma\sqrt{2}$ and σ is the width of the density of states (see details in Ref. [35]). For $M \sim 2N$ and for $M, N \gg 1$ one gets the estimate

$$t_S \sim N/\Gamma = N t_\Gamma. \quad (14)$$

This is the timescale for the complete thermalization in quantum MBSs. As one can see from Eq. (14), when the number of particles is very large, the two timescales are very different. Notice that for fixed Γ , the time t_S increases linearly with N due to the exponential growth with N of the many-body space and not because of the Gaussian shape of the density levels [35].

B. Results for the spin model. The analytical estimates obtained with the cascade approach are valid also for dynamical models. To show this, we study the evolution of the spin-1/2

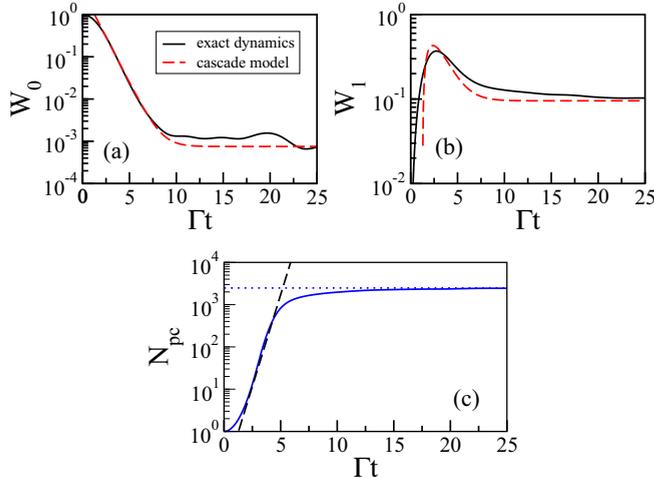


FIG. 2. Spin model: Numerical data (solid curves) for $W_0(t)$ (a) and $W_1(t)$ (b) compared with the analytical expressions (10) (dashed curves). (c) Numerical data for the number of principal components $N_{pc}(t)$ (solid curve) and the infinite-time average in Eq. (7) (dotted line). The dashed line represents $e^{2\Gamma t}$. Parameters: $L = 16$, $\Delta = 0.48$, $\lambda = 1$, and $N = 7$ excitations. Average over 16 initial states with energy close to -0.5 . Threshold for counting \mathcal{N}_1 is $\xi = 0.05$ and $\Gamma = 2.62$ is obtained by fitting $W_0(t)$.

system described by Eq. (3) in the limit of strong chaos ($\lambda = 1$) [22]. The analysis is analogous to the one developed with the TBRI model. We note, however, that H_0 is now initially written in the basis where each site has a spin pointing up or down in the z direction (site basis). It is then diagonalized to obtain the mean-field basis. As a result, all matrix elements of the full Hamiltonian written in the mean-field basis become nonzero. Therefore, to properly determine the classes, we use the following procedure. In the first class we have all states m coupled to k_0 such that $|H_{k_0,m}| > \xi |H_{k_0,k_0} - H_{m,m}|$ with ξ being a threshold reasonably chosen. This procedure is repeated for higher classes.

Figure 2 compares the numerical results for $W_0(t)$, $W_1(t)$, and $N_{pc}(t)$ for the spin model with the analytical expressions in Eqs. (10) and (11). The agreement is very good, and the exponential increase in time of the number of principal components with rate 2Γ is confirmed. As for the TBRI model, we see that the back flow is not important and that two classes suffice to describe the dynamics. This validates our approach for realistic physical systems even in the absence of any random parameter.

IV. Discussion. We studied the dynamics of interacting quantum MBSs whose eigenstates have a chaotic structure in the basis of noninteracting particles. We demonstrated that in the many-body space the relaxation is not a diffusive or ballistic process. Instead, wave packets evolve exponentially fast in the unperturbed basis before reaching saturation, which happens when all states of the energy shell get populated. Unexpectedly, we found that the timescale for saturation is much larger than the characteristic decay time of the initial state.

To describe the dynamical process, we developed a semi-analytical approach that allowed us to estimate the rate and

the timescale of the relaxation, as well as the saturation value of the number of principal components in the wave packet. It is quite impressive that our simple phenomenological model with a single parameter, the width Γ of LDOS, reproduces so well the system dynamics at very different timescales.

The first analytical investigation of the properties of the LDOS was done by Wigner in his studies of banded random matrices [38]. In the context of quantum chaos, these matrices were employed in Ref. [39], where it was pointed out that the LDOS has a well-defined classical limit and is the projection of the unperturbed Hamiltonian onto the total one. Its maximal width is given by the width of the energy shell, as shown in Ref. [39]. In the classical description, the energy shell corresponds to the phase-space volume obtained by the projection of the phase-space surface $H_0 = E_0$ onto the surface defined by the total Hamiltonian H . Note that the classical LDOS can be obtained by solving classical equations of motion [40]. The dynamics of the classical packets created by H_0 is restricted to the energy shell [40,41], which can be filled in time either partially or ergodically. In the quantum description, these two alternatives correspond to either localized or delocalized wave packets.

Inspired by the above studies, our results for the exponential growth of N_{pc} can be treated in terms of the phase-space volume \mathcal{V}_E occupied by the wave packet, $\mathcal{V}_E(t) \sim N_{pc}(t)/\rho(E)$, where $\rho(E)$ is the total density of states. We can write

$$\mathcal{V}_E(t) = \mathcal{V}_E(0)e^{2\Gamma t} \sim \mathcal{V}_E(0)e^{h_{KS}t}. \quad (15)$$

Here we associate 2Γ with the Kolmogorov-Sinai entropy [42], h_{KS} , which gives the exponential growth rate of phase-space volumes for classically chaotic MBSs [42]. A connection between the entanglement entropy growth rate and h_{KS} was found also in Ref. [43]. Note that in many-body systems, h_{KS} is defined as the sum of *all* positive Lyapunov exponents and not only the largest one. The relation $h_{KS} \sim 2\Gamma$ allows one to establish a quantum-classical correspondence for MBSs. Indeed, when the system admits a well-defined classical limit in which there is strong chaos, the Kolmogorov-Sinai entropy is associated with the width of the classical LDOS.

We stress that Eq. (15) holds only up to the saturation time $t_S \sim Nt_\Gamma$, which defines the timescale for the quantum-classical correspondence for the number of principal components N_{pc} participating in the dynamics. This time t_S is important for the problem of thermalization in isolated systems of interacting particles. It establishes the timescale for the complete thermalization of the system due to the ergodic filling of the energy shell. It also corresponds to the scrambling time discussed in studies of the loss of information in black holes (see Ref. [44] and references therein). One sees that in the thermodynamic limit, $N \rightarrow \infty$, t_S diverges (provided the width of the LDOS remains constant), which agrees with the quantum-classical correspondence principle.

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