# Ground-state energy distribution of disordered many-body quantum systems

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Extreme-value distributions are studied in the context of a broad range of problems, from the equilibrium properties of low-temperature disordered systems to the occurrence of natural disasters. Our focus here is on the ground-state energy distribution of disordered many-body quantum systems. We derive an analytical expression that, upon tuning a parameter, reproduces with high accuracy the ground-state energy distribution of all the systems that we consider. For several models with short- and long-range random couplings, the ground-state distribution agrees remarkably well with the Tracy-Widom distribution obtained from random matrices. They include transverse Ising models, the Sachdev-Ye model and variants, and a randomized version of the PXP model. In contrast, shapes at odds with the Tracy-Widom distribution are observed for disordered Bose-Hubbard models and the disordered spin-1/2 Heisenberg chain used to investigate many-body localization.

The level spacing distribution in the bulk of the spectrum of disordered many-body quantum systems and its comparison with full random matrices [1–4] has been intensively investigated, due to its relevance for the study of thermalization in isolated quantum systems [5, 6], many-body localization [7–10], and nonequilibrium quantum dynamics [11–15]. The present work focuses instead on the distribution of the lowest energy level of such systems and its relationship with random matrices, a subject that has received significantly less attention [16, 17].

Extreme-value statistics concerns the study of rare events, such as tsunamis, floods, earthquakes, and large variations in the stock market. It also involves the study of the fluctuations of the smallest (largest) eigenvalue of random matrices [18], which has applications in analyses of the stability of dynamical systems with interactions [19, 20] and of the equilibrium properties of disordered systems at low temperatures [20–22]. In the case of independent and identically distributed random variables, there are three universality classes for the distribution of the sample minimum (maximum). They are given by the Gumbel, Fréchet, or Weibull distribution, depending on the tail of the parent probability density of the variables [22]. However, when the random variables are correlated, there are very few cases for which the extreme-value distribution has been obtained [22], one being the distribution of the lowest (highest) eigenvalue of ensembles of large Gaussian random matrices derived by Tracy and Widom [23, 24].

The Tracy-Widom distribution arises in different theoretical and experimental contexts, such as in studies of mesoscopic fluctuations in quantum dots and of spatial correlations of non-interacting fermions at the edges of a trap (see [22] and references therein). Of particular interest to us is the verification that the ground-state energy distribution of even-even nuclei [25–27] agrees with the Tracy-Widom distribution obtained with Gaussian orthogonal ensembles (GOE) [28].

Nuclei are typical examples of interacting many-body quantum systems, so features found there may extend also to other many-body quantum systems. This prompts us to investigate in this work the distribution of the lowest level of different disordered many-body quantum systems.

We find excellent agreement with the GOE Tracy-Widom distribution for various spin models with short- and long-range random couplings, such as transverse Ising models [29], the Sachdev-Ye model [30], and a randomized version of the PXP model [31]. Many of these systems can be realized in experiments with cold atoms [32–34], ion traps [35, 36], and nuclear magnetic resonance [37, 38]. However, we also identify examples of disordered many-body quantum systems, where the ground-state energy distribution is other than the Tracy-Widom. These include spin-1/2 Heisenberg chains with onsite disorder and Bose-Hubbard models with random couplings. Using real-valued Wishart ensembles, we derive a general one-parameter analytical expression that reproduces up to high accuracy the ground-state energy distribution of all the models that we study, whether it is a Tracy-Widom distribution or not.

*Agreement with random matrices.*– The GOE consists of real-valued symmetric matrices with entries sampled independently from the Gaussian distribution with zero mean $\mu = 0$ and off-diagonal (diagonal) components with variance $\sigma^2 = 1/2$ ($\sigma^2 = 1$) [39, 40]. The eigenvalues $E_0, E_1, \dots, E_{N-1}$ are distributed according to

$$P(E_0, E_1, \dots, E_{N-1}) = \frac{1}{\mathcal{Z}_N} \prod_{i=0}^{N-1} e^{-E_i^2/2} \prod_{j<k} |E_j - E_k|, \tag{1}$$

where $\mathcal{Z}_N$ is a normalization constant that fixes the probability integral to unity. This expression does not factorize in terms that uniquely depend on a single eigenvalue, indicating that the eigenvalues are (strongly) correlated. Sorting the eigenvalues in increasing order, the distribution of the smallest eigenvalue $E_0$, is obtained by integrating out all other eigenvalues,

$$P(E_0) = \int P(E_0, E_1, \ldots, E_{N-1})\, dE_1\, dE_2 \ldots dE_{N-1}. \tag{2}$$

For $N \to \infty$, the distribution of the smallest eigenvalue converges towards the Tracy-Widom distribution [23, 24]. This distribution does not have a closed-form expression, but it can be written in terms of the solution of the Painlevé II differential equation and evaluated numerically. After some manipulation [41], one finds that the distribution of the ground-state energy has the shape

$$P(E_0) = \sqrt{F_2(-E_0)}\, \exp\left[\frac{1}{2}\int_{-E_0}^{\infty} q(x)dx\right] \tag{3}$$

with

$$F_2(x) = \exp\left[-\int_x^{\infty}(z-x)q^2(x)dz\right], \tag{4}$$

where $q(x)$ is the solution of the Painlevé II differential equation $q'' = xq + 2q^3$ subjected to the boundary condition $q(x) \sim \mathrm{Ai}(x)$ for $x \to \infty$, with $\mathrm{Ai}(x)$ denoting the Airy function.

We compare the Tracy-Widom distribution in Eq. (3) with the ground-state energy distribution of different disordered many-body quantum systems. For all the models considered in this work, the random parameters are independent real numbers sampled from the Gaussian distribution with mean $\mu = 0$ and variance $\sigma^2 = 1$. The ground-state energy distributions are shifted and scaled such that their mean and variance are zero and unity, respectively.

In Fig. 1 (a), we plot $P(E_0)$ for the Hamiltonian

$$H = \sum_{i=1}^{L} \frac{\epsilon_i}{2}\sigma_i^z + \sum_{i,j}^{L-1} \frac{J_{ij}}{4|j-i|^{\alpha}}\vec{\sigma}_i \cdot \vec{\sigma}_j, \tag{5}$$

where $L$ is the system size, $\epsilon_i$ and $J_{ij} = J_{ji}$ are random numbers, and $\sigma^{x,y,z}$ are the Pauli matrices. For all-to-all couplings ($\alpha = 0$), the Hamiltonian in Eq. (5) is analogous to the two-body random ensembles studied in quantum chaos and nuclear physics [1, 4, 42–44] and coincides with the Sachdev-Ye model when $\epsilon_i = 0$ [30]. We also study $\alpha > 0$, in which case we consider one-dimensional (1D) systems. Hamiltonian (5) conserves the total $z$-magnetization $\mathcal{S}_z = \sum_{i=1}^{L} S_i^z$.

The distribution of the ground-state energy of $H$ (5) is remarkably robust. As seen in Fig. 1 (a), the agreement with the Tracy-Widom distribution is excellent for $\alpha = 0$, and as $\alpha$ increases from all-to-all to short-range couplings, the deviations remain minor. For the $\mathcal{S}_z = 0$ sector, the agreement holds for system sizes as small as $L = 8$; and for large system sizes, there is very good agreement for as few as 4 excitations. $P(E_0)$ also matches the Tracy-Widom distribution for the Sachdev-Ye model, that is, for $H$ (5) with $\alpha = 0$ and $\epsilon_i = 0$.

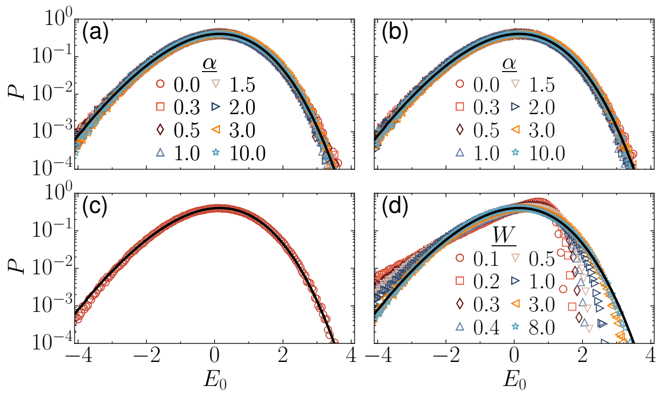


FIG. 1. Ground-state energy distribution (a)-(d) of the systems defined in Eqs. (5)-(8), respectively. The data (symbols) consist of $10^6$ realizations for (a) $H$ in Eq. (5) for the zero-magnetization sector ($\mathcal{S}_z = 0$) and system size $L = 16$, (b) $H$ in Eq. (6) for $L = 14$, (c) $H$ in Eq. (7) for $L = 16$, and (d) $H$ in Eq. (8) for $\mathcal{S}_z = 0$ and $L = 16$. The solid line represents the Tracy-Widom distribution.

The agreement with the Tracy-Widom distribution is also excellent in Fig. 1 (b) and Fig. 1 (c). In Fig. 1 (b), the comparison is made with the Ising model with longitudinal and transverse fields,

$$H = \sum_{i=1}^{L}\left(\frac{h_i^z}{2}\sigma_i^z + \frac{h_i^x}{2}\sigma_i^x\right) + \sum_{i,j}^{L-1} \frac{J_{ij}}{4|j-i|^{\alpha}}\sigma_i^z\sigma_j^z, \tag{6}$$

where $h_i^z$, $h_i^x$, and $J_{ij} = J_{ji}$ are random numbers. In the case of nearest-neighbor couplings, Eq. (6) agrees with the Hamiltonian used to study quantum phase transitions in Ref. [29]. We examine both short- and long-range interactions. In Fig. 1 (c), we consider the Hamiltonian

$$H = \sum_{i=1}^{L-2} J_{i+1}P_i\sigma_{i+1}^x P_{i+2} + J_1\sigma_1^x P_2 + J_L P_{L-1}\sigma_L^x, \tag{7}$$

where $P_i = (1-\sigma_i^z)/2$ denotes the projection operator and $J_i$ are random numbers. The main mechanism of this model is based on local constraints that forbid two adjacent spins to be simultaneously in the up-state. For $J_i = 1$, the Hamiltonian reduces to the PXP model [31, 32], which is paradigmatic in studies on quantum many-body scars and weak ergodicity breaking [45, 46]. For random parameters, model (7) is similar to the one introduced in [47].

Contrary to the previous models, the ground-state energy distribution of the disordered spin-1/2 Heisenberg Hamiltonian employed in studies of many-body localization [7–10],

$$H = W\sum_{i=1}^{L}\frac{h_i}{2}\sigma_i^z + \frac{J}{4}\sum_{i=1}^{L-1}\vec{\sigma}_i \cdot \vec{\sigma}_{i+1}, \tag{8}$$

does not always agree with the Tracy-Widom distribution, as seen in Fig. 1 (d). In the equation above, $h_i$

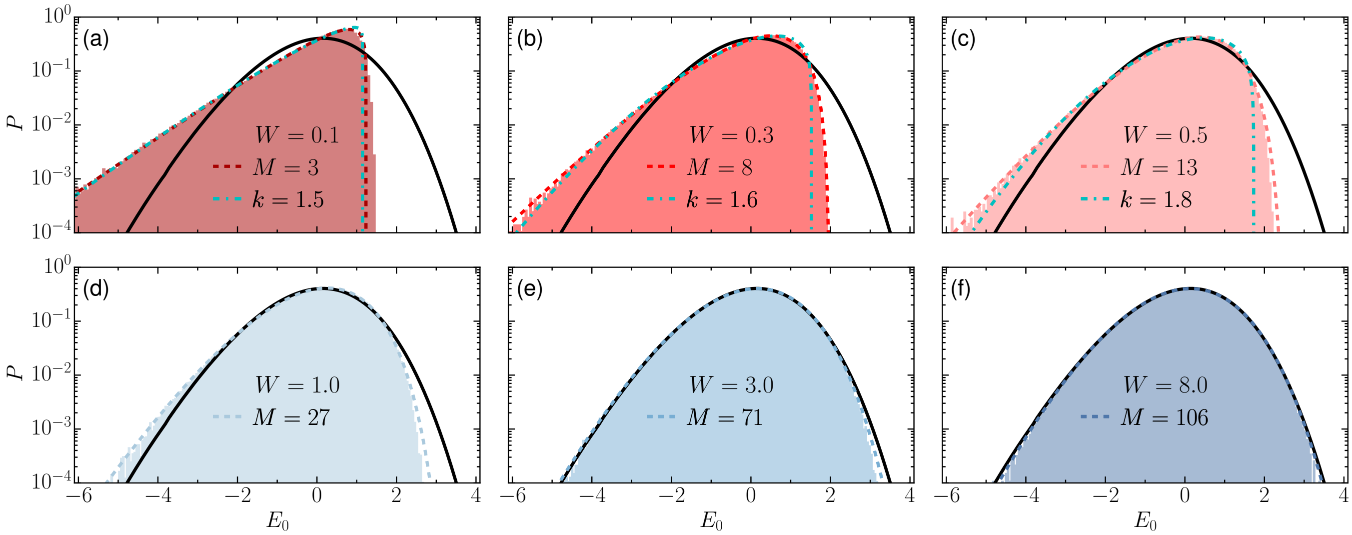


FIG. 2. Ground-state energy distribution of the Hamiltonian in Eq. (8), which is prototypical in studies of many-body localization. The distribution deviates from (approaches) the Tracy-Widom distribution for disorder strength $0 < W \leq J$ ($W > J$). The data (shades) are obtained using $10^6$ realizations for the zero-magnetization sector ($\mathcal{S}_z = 0$) and system size $L = 16$. Solid, dashed, and dot-dashed lines represent the Tracy-Widom, $\chi^2$-, and Weibull distribution, respectively. The values of $M$ are rounded to their closest integers.

are random numbers, $W$ is the disorder strength, and the coupling parameter $J = 1$. The properties of the bulk of the spectrum of this model are similar to those of GOE random matrices when $0 < W \leq J$, but differ when $W > J$ [48, 49]. In stark contrast with the bulk of the spectrum, the ground-state energy distribution of $H$ (8) differs from the GOE Tracy-Widom when $0 < W \leq J$ and approaches it for $W > J$. To identify the distributions that best match those shown in Fig. 1 (d), we resort to real-valued Wishart ensembles, as described below.

*General analytical expression.–* The Wishart ensemble is one of the classical ensembles in random matrix theory and precedes the Gaussian ensembles [40]. It consists of matrices $A = XX^T$, where $X$ is an $N \times M$ matrix with (real-valued) entries sampled independently from the Gaussian distribution with mean $\mu = 0$ and variance $\sigma^2 = 1$. The eigenvalues $E_0, E_1, \ldots, E_{N-1}$ are distributed according to

$$P(E_0, E_1, \ldots, E_{N-1}) = \frac{1}{\mathcal{Z}_N} \prod_{i=0}^{N-1} e^{-E_i/2} \times \prod_i E_i^{(M-N-1)/2} \prod_{j<k} |E_j - E_k|, \quad (E_i \geq 0), \tag{9}$$

where, as in Eq. (1), the prefactor $1/\mathcal{Z}_N$ is a normalization constant fixing the integral probability to unity.

For $N = 1$, we find that by varying $M$, Eq. (9) can capture the various levels of asymmetry exhibited by the ground-state energy distributions seen in Fig. 1 (d). In this case, the Wishart ensemble consists of samples given by the sum of $M$ squared independent random numbers, so Eq. (9) reduces to the chi-squared ($\chi^2$-) distribution,

$$P(E_0) = \frac{1}{2^{M/2}\Gamma(M/2)} e^{-E_0/2} E_0^{M/2-1}, \quad (E_0 \geq 0), \tag{10}$$

with $M$ degrees of freedom, mean $\mu = M$, and variance $\sigma^2 = 2M$. Since our focus is on minima, instead of maxima, we consider the distribution of $-E_0$.

To find the best-fitting value of $M$ for a given ground-state energy distribution, we use the skewness $\gamma = -\mu_3/\mu_2^{3/2}$ as a matching parameter, taking into account that the $n$-th moment is $\mu_n = \int (x-\mu)^n P(x)\, dx$. For the $N = 1$ Wishart ensemble, $\gamma = -2\sqrt{2/M}$ and thus

$$M = \frac{8}{\gamma^2}. \tag{11}$$

*Agreement with the $\chi^2$-distribution.–* In Fig. 2, we select some of the curves exhibited in Fig. 1 (d) to show that the analytical expression in Eq. (10) matches the numerical results for $P(E_0)$ obtained with $H$ (8) for any disorder strength $W$.

The best-fitting $\chi^2$-distributions for the cases with small disorder strength ($W < 1$) resemble the Weibull distribution,

$$P(E_0) = \frac{k}{\lambda} \left( \frac{E_0 - \theta}{\lambda} \right)^{k-1} e^{-\left(\frac{E_0 - \theta}{\lambda}\right)^k}, \qquad (E_0 > \theta). \tag{12}$$

The parameters $\theta$, $\lambda$, and $k$ are computed to ensure that the distribution has zero mean, unit variance, and the same skewness as that of the numerical data in

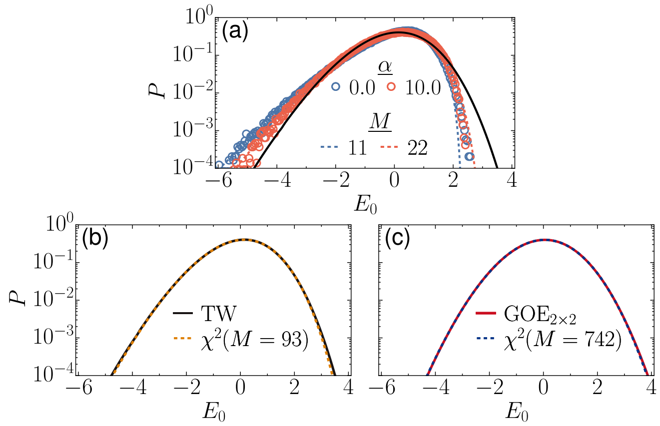


FIG. 3. In (a): The ground-state energy distribution for $H$ (13) with $N_b = L = 7$. Symbols are numerical data, dashed lines represent the $\chi^2$-distribution, and the solid line is the Tracy-Widom distribution. Comparison between the Tracy-Widom [$2 \times 2$ GOE ground-state energy] distribution and the $\chi^2$-distribution is shown in (b) [(c)].

Figs. 2 (a)-(c). As $W$ increases above 1, the best-fitting $\chi^2$-distributions approach the Tracy-Widom distribution, the agreement being very good when $W = 8$ in Fig. 2 (f).

For models that, similarly to $H$ (8), have a fixed constant parameter, such as the Hamiltonians in [47, 50], we verify that the ground-state energy distribution also deviates from the Tracy-Widom distribution. For all of these cases, $P(E_0)$ can be reproduced with an appropriate choice of $M$ in Eq. (10). This equation also holds for the disordered 1D Bose-Hubbard model,

$$H = \sum_{i=1}^{L} \frac{U_i}{2} n_i (n_i - 1) - \sum_{i,j}^{L-1} \frac{J_{i,j}}{|j-i|^{\alpha}} \left( a_i^{\dagger} a_j + a_j^{\dagger} a_i \right), \tag{13}$$

where $a_i$ ($a_i^{\dagger}$) is the annihilation (creation) operator, $n_i = a_i^{\dagger} a_i$, and the parameters $U_i$ and $J_{i,j}$ are random numbers. When the total number of bosons, $N_b$, is small, $P(E_0)$ shows good agreement with the Tracy-Widom distribution. However, for $N_b = L$, deviations occur for both short- and long-range couplings, as seen in Fig. 3 (a), and for any value of $\alpha$, the shapes are well captured by the $\chi^2$-distribution.

A natural question that emerges from our comparisons between Eq. (10) and the ground-state energy distribution of various disordered models is the value of $M$ that leads to the agreement between the Tracy-Widom distribution and the $\chi^2$-distribution. The skewness of the GOE Tracy-Widom distribution has been obtained numerically, $\gamma = -0.2934645240\ldots$ [41]. Equating this value to the skewness of the $\chi^2$-distribution gives $M = 93$ after rounding to the nearest integer. As shown in Fig. 3 (b), the two curves agree down to very small values. The parallel between these two distributions is useful, because a closed form for the Tracy-Widom distribution does not yet exist. One needs to evaluate it numerically, which requires solving the Painlevé II differential equation. Using instead the $\chi^2$-distribution is a simple and accurate alternative (see also Ref. [51]).

By further increasing $M$ beyond the Tracy-Widom shape, the $\chi^2$-distribution eventually matches the analytical expression of $P(E_0)$ obtained from $2 \times 2$ GOE random matrices,

$$P(E_0) = \frac{1}{2\sqrt{\pi}} e^{-E_0^2} - \frac{1}{2\sqrt{2}} e^{-\frac{1}{2}E_0^2} E_0 \,\text{erfc}\left(\frac{E_0}{\sqrt{2}}\right), \tag{14}$$

which, in turn, is comparable to the ground-state energy distribution of some nuclear and molecular models [28]. The skewness of the distribution in Eq. (14), $\gamma = -[2(\pi - 3)\sqrt{\pi}/(6-\pi)^{3/2}]$, matches the skewness of the $\chi^2$-distribution for $M = 742$ after rounding to the nearest integer. The excellent agreement between the two curves is illustrated in Fig. 3 (c). Finally, for $M \to \infty$, the $\chi^2$-distribution converges towards a Gaussian.

*Conclusions.*– The use of the $\chi^2$-distribution to study the lowest energy level distribution of quantum systems is analogous to the use of the Brody distribution [1] (or the Izrailev distribution [52]) to study the level spacing distribution of the bulk of their spectra. By varying a fitting parameter, the Brody distribution changes from Poisson to Wigner-Dyson, covering all shapes in between. Here, we show that by varying the number of degrees of freedom of the $\chi^2$-distribution, we can reproduce with great accuracy the shape of the ground-state energy distribution for any of the disordered many-body quantum systems that we consider. In the case of the Heisenberg spin-1/2 chain with onsite disorder, in particular, the $\chi^2$-distribution captures the crossover from the Weibull distribution to the Tracy-Widom distribution as the disordered strength increases.

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